ECS 452: In-Class ~~Exercise # _~~ Additional Example

Instructions

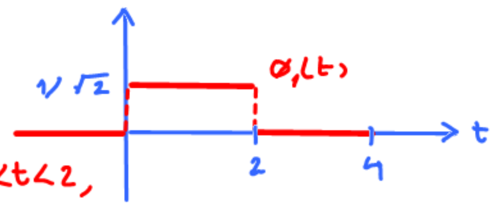
1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups after the midterm.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: _ _ / _ _ / 2018			
Name	ID (last 3 digits)		

Continue from Exercise #18.

d. Suppose the Gram-Schmidt orthogonalization procedure (GSOP) (where the signals are applied **in the order given**) is used to find two orthonormal functions $\phi_1(t)$ and $\phi_2(t)$ that can be used as axes to represent $s_1(t)$ and $s_2(t)$.

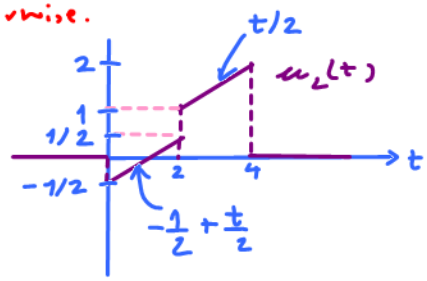
i. Find and plot $\phi_1(t)$



$u_1(t) = s_1(t)$ $\qquad s_1 = 2\sqrt{2}\,\phi_1 \approx 2.83\phi_1$

$$\phi_1(t) = \frac{u_1(t)}{\sqrt{E_{u_1}}} = \frac{s_1(t)}{\sqrt{E_1}} = \frac{1}{2\sqrt{2}} s_1 = \begin{cases} 1/\sqrt{2}, & 0<t<2, \\ 0, & \text{otherwise.} \end{cases}$$

ii. Find and plot $\phi_2(t)$.

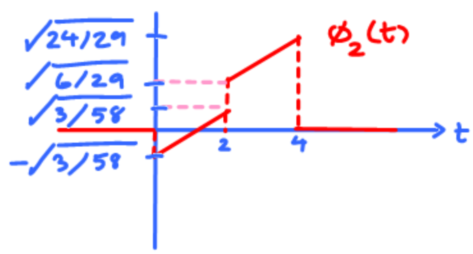


$$u_2(t) = s_2(t) - \text{proj}_{u_1} s_2 = s_2(t) - \frac{1}{4}s_1(t)$$

$$E_{u_2} = \int_{-\infty}^{\infty} u_2^2(t)\,dt = \int_0^2 \left(-\frac{1}{2}+\frac{t}{2}\right)^2 dt + \int_2^4 \left(\frac{t}{2}\right)^2 dt$$

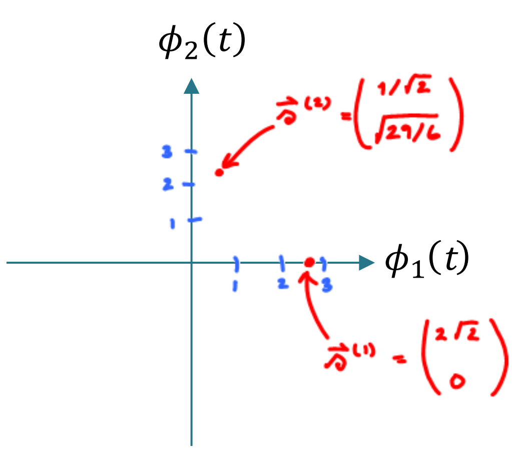
$$= \frac{1}{6} + \frac{14}{3} = \frac{1+28}{6} = \frac{29}{6}$$

$$\phi_2(t) = \frac{u_2(t)}{\sqrt{E_{u_2}}} = \sqrt{\frac{6}{29}}\,u_2(t) = \begin{cases} \sqrt{\frac{3}{58}}(t-1), & 0<t<2, \\ \sqrt{\frac{3}{58}}\,t, & 2\le t<4, \\ 0, & \text{otherwise.} \end{cases}$$



$$s_2(t) = \frac{1}{4}s_1(t) + u_2(t) = \frac{1}{4}2\sqrt{2}\,\phi_1(t) + \sqrt{\frac{29}{6}}\,\phi_2(t) = \frac{1}{\sqrt{2}}\phi_1(t) + \sqrt{\frac{29}{6}}\,\phi_2(t) \approx 0.71\,\phi_1(t) + 2.2\,\phi_2(t)$$

iii. Find the two vectors, $\vec{\mathbf{s}}^{(1)}$ and $\vec{\mathbf{s}}^{(2)}$, that represent the two waveforms $s_1(t)$ and $s_2(t)$ in the new axes based on $\phi_1(t)$ and $\phi_2(t)$. Draw the corresponding constellation in the figure below.



Double-checking:

$E_1 = 2^2 \times 2 = 4 \times 2 = 8$

$E_2 = \frac{1}{2} + \frac{29}{6} = \frac{3}{6} + \frac{29}{6} = \frac{32}{6} = \frac{16}{3}$

$\langle s_1, s_2 \rangle = 2\sqrt{2} \times \frac{1}{\sqrt{2}} = 2$