ECS 452: In-Class Exercise #17

Date: 27 / 04 / 2018	
Name	ID (last 3 digits)
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Instructions

1. Separate into groups of no more than three persons. The group cannot be the same as any of your former groups after the midterm.
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

1. Suppose $\vec{\mathbf{v}} = \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix}$ and $\vec{\mathbf{u}} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$.

   a. Find $\langle \vec{\mathbf{v}}, \vec{\mathbf{u}} \rangle = (4)(2) + (0)(1) + (-3)(-2) = 8 + 0 + 6 = 14$

   For $\vec{v} = (v_1, v_2, v_3)^T$ and $\vec{u} = (u_1, u_2, u_3)^T$, we have $\langle \vec{v}, \vec{u} \rangle = v_1 u_1 + v_2 u_2 + v_3 u_3$

   b. Find $\langle \vec{\mathbf{u}}, \vec{\mathbf{u}} \rangle = 2^2 + 1^2 + (-2)^2 = 4 + 1 + 4 = 9$

   For $\vec{u} = (u_1, u_2, u_3)^T$, we have $\langle \vec{u}, \vec{u} \rangle = u_1 u_1 + u_2 u_2 + u_3 u_3 = u_1^2 + u_2^2 + u_3^2$

   c. Find $\langle \vec{\mathbf{v}}, \vec{\mathbf{v}} \rangle = 4^2 + 0^2 + (-3)^2 = 16 + 9 = 25$

   Similar to part (b), for $\vec{v} = (v_1, v_2, v_3)^T$, we have $\langle \vec{v}, \vec{v} \rangle = v_1^2 + v_2^2 + v_3^2$

   d. Find $\|\vec{\mathbf{v}}\| \equiv \sqrt{\langle \vec{v}, \vec{v} \rangle} = \sqrt{25} = 5$

   by definition

   e. Find $\|\vec{\mathbf{u}}\| \equiv \sqrt{\langle \vec{u}, \vec{u} \rangle} = \sqrt{9} = 3$

   f. Find $\text{proj}_{\vec{\mathbf{u}}} \vec{\mathbf{v}} \equiv \frac{\langle \vec{v}, \vec{u} \rangle}{\langle \vec{u}, \vec{u} \rangle} \vec{u} = \frac{14}{9} \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 28/9 \\ 14/9 \\ -28/9 \end{pmatrix} \approx \begin{pmatrix} 3.1111 \\ 1.5556 \\ -3.1111 \end{pmatrix}$

   by definition

   Direct calculation:

   $\vec{z} = (4, 0, -3)^T - \frac{1}{9}(28, 14, -28)^T = \frac{1}{9}(8, -14, 1)^T$

   $\langle \vec{u}, \vec{z} \rangle = \frac{1}{9}\big((2)(8) + (1)(-14) + (-2)(1)\big) = \frac{1}{9} \times 0 = 0.$

   g. Let $\vec{\mathbf{z}} = \vec{\mathbf{v}} - \text{proj}_{\vec{\mathbf{u}}} \vec{\mathbf{v}}$. Find $\langle \vec{\mathbf{u}}, \vec{\mathbf{z}} \rangle$.

   We know that $\underbrace{\vec{p} = \text{proj}_{\vec{u}} \vec{v}}_{\vec{u} \text{ is parallel to } \vec{p}}$ and $\underbrace{\vec{o} = \vec{v} - \vec{p}}_{\vec{z} \text{ is exactly the same as } \vec{o}}$ are always orthogonal.

   Therefore, $\vec{u}$ and $\vec{z}$ are orthogonal and hence $\langle \vec{u}, \vec{z} \rangle = 0$.

   Alternatively,

   $$\langle \vec{u}, \vec{z} \rangle = \langle \vec{u}, \vec{v} - \text{proj}_{\vec{u}} \vec{v} \rangle = \langle \vec{u}, \vec{v} \rangle - \left\langle \vec{u}, \frac{\langle \vec{v}, \vec{u} \rangle}{\langle \vec{u}, \vec{u} \rangle} \vec{u} \right\rangle = \langle \vec{u}, \vec{v} \rangle - \frac{\cancel{\langle \vec{u}, \vec{u} \rangle}}{\cancel{\langle \vec{u}, \vec{u} \rangle}} \langle \vec{v}, \hat{u} \rangle = 0$$