ECS 452: In-Class Exercise # 14

Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups after the midterm.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: 10 /04 / 2018			
Name	ID (last 3 digits)		
Prapun	**5**	**5**	**5**

Consider a block code whose generator matrix is

$$\mathbf{G} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{I_3} \underbrace{\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}}_{P}$$

a. Find the parity check matrix **H** of this code.

$$H = \left[\underbrace{\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}}_{P^T} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{I} \right]$$

b. Suppose we receive $\underline{\mathbf{y}}$ = 111001

i. Find the syndrome vector $\underline{s}$

$$\underline{s} = \underline{y} H^T = \left[\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right]^T = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}^T = [0\,0\,1]$$

ii. Find the decoded codeword $\hat{\underline{x}}$

The syndrome $\underline{s}$ is the same as the last column of H.

Therefore, $\hat{\underline{e}} = [0\,0\,0\,0\,0\,1]$ and

$$\hat{\underline{x}} = \hat{\underline{y}} - \hat{\underline{e}} = \hat{\underline{y}} \oplus \hat{\underline{e}} = [1\,1\,1\,0\,0\,0]$$

iii. Find the decoded message $\hat{\underline{b}}$.

$\hat{\underline{b}} = [1\,1\,1]$

From G, we have I_3 in the front, so the message $\underline{b}$ will be the first three bits of the codeword $\underline{x}$.