ECS 452: In-Class Exercise #13

Instructions

1. Separate into groups of no more than three persons.
2. The group cannot be the same as your former group.
3. Only one submission is needed for each group.
4. ***Write down all the steps*** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
5. **Do not panic.**

Date: 03 / 04 / 2017			
Name	ID (last 3 digits)		
Prapun	5	5	5

1. Consider a linear block code that uses parity checking on a square array:

b_1	b_2	$p_2 = b_1 \oplus b_2$
b_3	b_4	$p_3 = b_3 \oplus b_4$
$b_1 \oplus b_3 = p_1$	$p_4 = b_2 \oplus b_4$	

Each parity bit p_i is calculated such that the corresponding row or column has even parity.

Suppose the following bits arrangement is used in the codeword:

$$\underline{\mathbf{x}} = \left(b_1 \quad p_1 \quad p_2 \quad b_2 \quad b_3 \quad b_4 \quad p_3 \quad p_4\right).$$

a. Find the generator matrix $\mathbf{G}$.

$$G = \begin{bmatrix} 1&1&1&0&0&0&0&0 \\ 0&0&1&1&0&0&0&1 \\ 0&1&0&0&1&0&1&0 \\ 0&0&0&0&0&1&1&1 \end{bmatrix}$$

Alternatively, use the square array:

① put $\underline{b}$ inside the square

$$\begin{array}{cccc} 0 & 1 & \xrightarrow{\Sigma} & 1 \\ 1 & 0 & \xrightarrow{\Sigma} & 1 \\ \Sigma\downarrow & \downarrow\Sigma & & \\ 1 & 1 & & \end{array}$$

② calculate the parity bits

③ put the values in the corresponding positions

0 1 1 1 1 0 1 1

b. Find the codeword for the message $\underline{\mathbf{b}} = \begin{bmatrix} 0 & 1 & 1 & 0 \end{bmatrix}$.

$$\underline{x} = \underline{b}G = [0\ 1\ 1\ 0]\begin{bmatrix} g^{(1)} \\ g^{(2)} \\ g^{(3)} \\ g^{(4)} \end{bmatrix} = g^{(2)} \oplus g^{(3)} = \begin{array}{r} 0\,0\,1\,1\,0\,0\,0\,1 \\ \oplus\; 0\,1\,0\,0\,1\,0\,1\,0 \\ \hline 0\,1\,1\,1\,1\,0\,1\,1 \end{array}$$

c. Find the parity-check matrix $\mathbf{H}$.

Identity matrix in the data positions becomes identity matrix in the parity positions

$$G = \begin{bmatrix} 1&1&1&0&0&0&0&0 \\ 0&0&1&1&0&0&0&1 \\ 0&1&0&0&1&0&1&0 \\ 0&0&0&0&0&1&1&1 \end{bmatrix} \qquad \begin{bmatrix} 1&1&0&0&1&0&0&0 \\ 1&0&1&1&0&0&0&0 \\ 0&0&0&0&1&1&1&0 \\ 0&0&0&1&0&1&0&1 \end{bmatrix} = H$$

Bit values in the parity positions are transposed and put in the data positions