ECS 452: In-Class Exercise # 12.1

Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups after the midterm.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: **30** / **03** / 2018			
Name	ID (last 3 digits)		
Prapun	**5**	**5**	**5**

1. A linear block code uses the following generator matrix $\mathbf{G} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$. $\begin{matrix} \rightarrow \underline{g}^{(1)} \\ \rightarrow \underline{g}^{(2)} \end{matrix}$

   a. Find the code length n **The generator marix has 4 columns.** **n = 4**

   b. Find the codeword for the message $\underline{\mathbf{b}} = [0 \quad 1]$

   $$\underline{x} = \underline{b}\,G = [0 \quad 1]\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} = [1 \quad 0 \quad 0 \quad 1]$$

   Direct multiplication like this is OK when we only have to find one codeword. However, in the next part, we need to find all of them; so we will use other methods.

   c. Find the codebook for this code.

   Method : $\underline{x} = \sum_i b_i \underline{g}^{(i)}$ where b_i = the i^{th} element in $\underline{b}$ and $\underline{g}^{(i)}$ = the i^{th} row of G

$\underline{\mathbf{b}}$	$\underline{\mathbf{x}}$	
00	0000	$\leftarrow 0\cdot\underline{g}^{(1)} \oplus 0\cdot\underline{g}^{(2)} = \underline{0}$
01	1001	$\leftarrow 0\cdot\underline{g}^{(1)} \oplus 1\cdot\underline{g}^{(2)} = \underline{g}^{(2)}$
10	0111	$\leftarrow 1\cdot\underline{g}^{(1)} \oplus 0\cdot\underline{g}^{(2)} = \underline{g}^{(1)}$
11	1110	$\leftarrow 1\cdot\underline{g}^{(1)} \oplus 1\cdot\underline{g}^{(2)} = \underline{g}^{(1)} \oplus \underline{g}^{(2)}$

   Method 2: $\underline{b} = [b_1 \quad b_2]$

   $$\underline{x} = \underline{b}\,G = [b_2 \quad b_1 \quad b_1 \quad b_1 \oplus b_2]$$

   The first bit is simply b_2.

   The second and third bits are the same as b_1.

   The last bit is the sum of b_1 and b_2.

2. Each row of the table below correspond to a code that uses **single-parity-check**. The error pattern is given.
   Find the corresponding values of codeword length n, code dimension k, and Indicate whether the given error pattern is detectable.

   # 1s is odd   # 1s is even

Error Pattern	codeword length n	code dimension k	$\underline{\mathbf{e}}$ is detectable (Yes or No?)
$\underline{\mathbf{e}} = [1 \quad 0 \quad 0 \quad 1]$	4	3	No
$\underline{\mathbf{e}} = [1 \quad 0 \quad 1 \quad 0 \quad 1]$	5	4	Yes
$\underline{\mathbf{e}} = [0 \quad 0 \quad 0 \quad 0 \quad 1]$	5	4	Yes
$\underline{\mathbf{e}} = [1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0]$	7	6	No

The length of vectors $\underline{e}, \underline{x}, \underline{y}$ should all be the same. $(\underline{y} = \underline{x} \oplus \underline{e})$

So, n = length of $\underline{e}$

For single-parity-check code, $k = n - 1$