

ECS 452: In-Class Exercise # 11

Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups.** Only one submission is needed for each group.
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: **20/03/2018**

Name

ID (last 3 digits)

Prapun

5 5 5

1. Assume GF(2). Calculate the following quantities:

a. $1 \oplus 1 = 0$

b. $1 \oplus 1 \oplus 1 = (1 \oplus 1) \oplus 1 = 0 \oplus 1 = 1$

c. $1 \cdot 0 = 0$

d. $1 \cdot 0 \cdot 1 = 0$

e. $[1 \ 1 \ 1] \oplus [1 \ 0 \ 1] = [1 \oplus 1 \quad 1 \oplus 0 \quad 1 \oplus 1] = [0 \ 1 \ 0]$

f. $[0 \ 1 \ 1] \oplus [0 \ 1 \ 1] = [0 \oplus 0 \quad 1 \oplus 1 \quad 1 \oplus 1] = [0 \ 0 \ 0]$ Fact: $x \oplus x = 0$

g. $[1 \ 0 \ 1] \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} = [(1 \cdot 1) \oplus (0 \cdot 0) \oplus (1 \cdot 1) \quad (1 \cdot 0) \oplus (0 \cdot 1) \oplus (1 \cdot 0)] = [0 \ 0]$

Alternatively, $[1 \ 0 \ 1] \begin{bmatrix} r^{(1)} \\ r^{(2)} \\ r^{(3)} \end{bmatrix} = r^{(1)} \oplus r^{(3)} = [1 \ 0] \oplus [1 \ 0] = [0 \ 0]$ *Block matrix multiplication*

2. A codeword $[1 \ 1 \ 0 \ 1]$ is sent over the BSC. Suppose the error pattern is $e = [0 \ 0 \ 1 \ 1]$.

Find the observed vector at the receiver.

$y = x \oplus e = [1 \ 1 \ 0 \ 1] \oplus [0 \ 0 \ 1 \ 1] = [1 \ 1 \ 1 \ 0]$

Alternatively, $e = [0 \ 0 \ 1 \ 1]$ means that the last two bits of x are received in error.

3. A codeword $[1 \ 1 \ 0 \ 1]$ is sent over the BSC. Suppose the observed vector at the receiver is

$y = [0 \ 1 \ 1 \ 1]$. Find the error pattern.

$y = x \oplus e$

$x \oplus y = x \oplus (x \oplus e) = x \oplus x \oplus e = e$

$e = x \oplus y = [1 \ 1 \ 0 \ 1] \oplus [0 \ 1 \ 1 \ 1]$

Alternatively, the error pattern indicates the locations of errors in the observed vector, y .

4. A codeword is sent over the BSC.

Suppose the observed vector at the receiver is $y = [0 \ 1 \ 1 \ 1]$ and the error pattern is $e = [0 \ 0 \ 1 \ 1]$. x and

Find the transmitted codeword.

$y = x \oplus e$

$x \oplus e = x \oplus e \oplus e$

$x = y \oplus e = [0 \ 1 \ 1 \ 1] \oplus [0 \ 0 \ 1 \ 1]$

Alternatively, the error pattern says that the last two bits in the observed vector are received incorrectly. So, we need to flip their values to get the transmitted codeword.