ECS 452: Digital Communication Systems **2016/2**

HW 9 — Due: May 23

Lecturer: Asst. Prof. Dr. Prapun Suksompong

Instructions

(a) This assignment has 7 pages.

(b) (1 pt) Write your first name and the last three digit of your student ID on the upper-right corner of ***every*** submitted sheet.

(c) (1 pt) For each part, write your explanation/derivation and answer in the space provided.

(d) (8 pt) It is important that you try to solve all non-optional problems.

(e) Late submission will be heavily penalized.

Problem 1. Consider the two signals $s_1(t)$ and $s_2(t)$ shown in Figure 9.1. Note that V and T_b are some positive constants. Your answers should be given in terms of them.

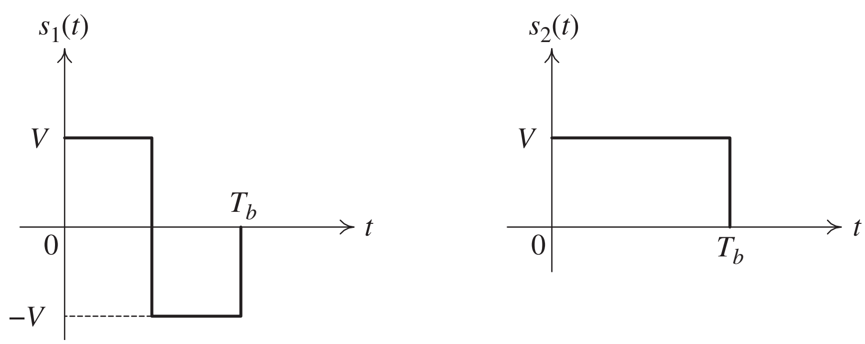

Figure 9.1: Signal set for Question 1

(a) Find the energy in each signal.

(b) Use the Gram-Schmidt orthogonalization procedure (GSOP) (where the signals are applied **in the order given**) to find two orthonormal functions $\phi_1(t)$ and $\phi_2(t)$ that can be used to represent $s_1(t)$ and $s_2(t)$.

(c) Find the two vectors that represent the two waveforms $s_1(t)$ and $s_2(t)$ in the new (signal) space based on the orthonormal basis found in the previous part. Draw the corresponding constellation.

Problem 2. Consider the two signals $s_1(t)$ and $s_2(t)$ shown in Figure 9.2. Note that V, α and T_b are some positive constants.

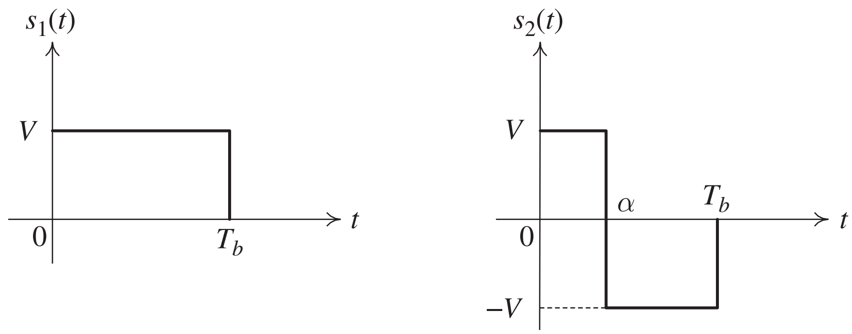

Figure 9.2: Signal set for Question 2

(a) Find the energy in each signal.

(b) Use the Gram-Schmidt orthogonalization procedure (GSOP) (where the signals are applied **in the order given**) to find two orthonormal functions $\phi_1(t)$ and $\phi_2(t)$ that can be used to represent $s_1(t)$ and $s_2(t)$.

(c) Plot $\phi_1(t)$ and $\phi_2(t)$ when $\alpha = \frac{T_b}{4}$.

(d) Find the two vectors $\mathbf{s}^{(1)}$ and $\mathbf{s}^{(2)}$ that represent the two waveforms $s_1(t)$ and $s_2(t)$ in the new (signal) space based on the orthonormal basis found in the previous part.

(e) Draw the corresponding constellation when $\alpha = \frac{T_b}{4}$.

(f) Draw $\mathbf{s}^{(2)}$ when $\alpha = \frac{k}{10}T_b$ where $k = 1, 2, \dots, 9$.

Problem 3. In a ternary signaling scheme, the message S is randomly selected from the alphabet set $\mathcal{S} = \{-1, 1, 4\}$ with $p_1 = P[S = -1] = 0.41$, $p_2 = P[S = 1] = 0.08$ and $p_3 = P[S = 4] = 0.51$. Find the average signal energy E_s.

Problem 4. Consider a ternary constellation. Assume that the three vectors are equiprobable.

(a) Suppose the three vectors are

$$\mathbf{s}^{(1)} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{s}^{(2)} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, \text{ and } \mathbf{s}^{(3)} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

Find the corresponding average energy per symbol.

(b) Suppose we can shift the above constellation to other location; that is, suppose that the three vectors in the constellation are

$$\mathbf{s}^{(1)} = \begin{pmatrix} 0 - a_1 \\ 0 - a_2 \end{pmatrix}, \mathbf{s}^{(2)} = \begin{pmatrix} 3 - a_1 \\ 0 - a_2 \end{pmatrix}, \text{ and } \mathbf{s}^{(3)} = \begin{pmatrix} 3 - a_1 \\ 3 - a_2 \end{pmatrix}.$$

Find a_1 and a_2 such that corresponding average energy per symbol is minimum.

Problem 5. (Optional) Suppose $s_1(t) = \text{sinc}(5t)$ and $s_2(t) = \text{sinc}(7t)$. Note that in this class, we define $\text{sinc}(x) = \frac{\sin x}{x}$. Find

(a) E_{s_1},

(b) E_{s_2}, and

(c) $\langle s_1(t), s_2(t) \rangle$.

Hint: Use Parseval's theorem to evaluate the above quantities in the frequency domain. (Review: See p. 21 of ECS332 lecture notes and Problem 4 in ECS332 HW3.)

Problem 6. (Optional) Prove the following facts with the help of Fourier transform. (Hint: inner product in the frequency domain, Parseval's theorem)

(a) The energy of $p(t) = g(t)\cos(2\pi f_c t + \phi)$ is $E_g/2$.

(b) $g(t)\cos(2\pi f_c t)$ and $-g(t)\sin(2\pi f_c t)$ are orthogonal.

Is there any condition(s) on $g(t)$ for this technique to work?