ECS 452: Digital Communication Systems **2016/2**

HW 8 — Due: May 16

Lecturer: Asst. Prof. Dr. Prapun Suksompong

Instructions

(a) This assignment has 4 pages.

(b) (1 pt) Write your first name and the last three digit of your student ID on the upper-right corner of ***every*** submitted sheet.

(c) (1 pt) For each part, write your explanation/derivation and answer in the space provided.

(d) (8 pt) It is important that you try to solve all non-optional problems.

(e) Late submission will be heavily penalized.

Problem 1. Consider a convolutional encoder whose circuit diagram and a part of the corresponding state diagram is given in Figure 8.1. Write the suitable labels for the two arrows shown in the state diagram.

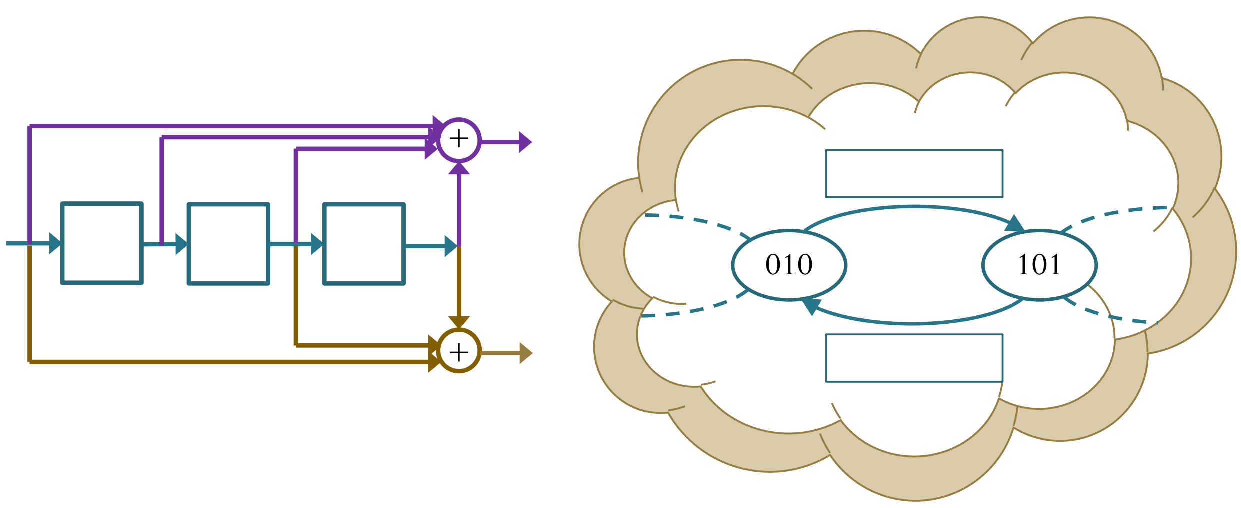

Figure 8.1: Circuit diagram and a part of the corresponding state diagram for a convolutional encoder. Only two states are shown in the state diagram.

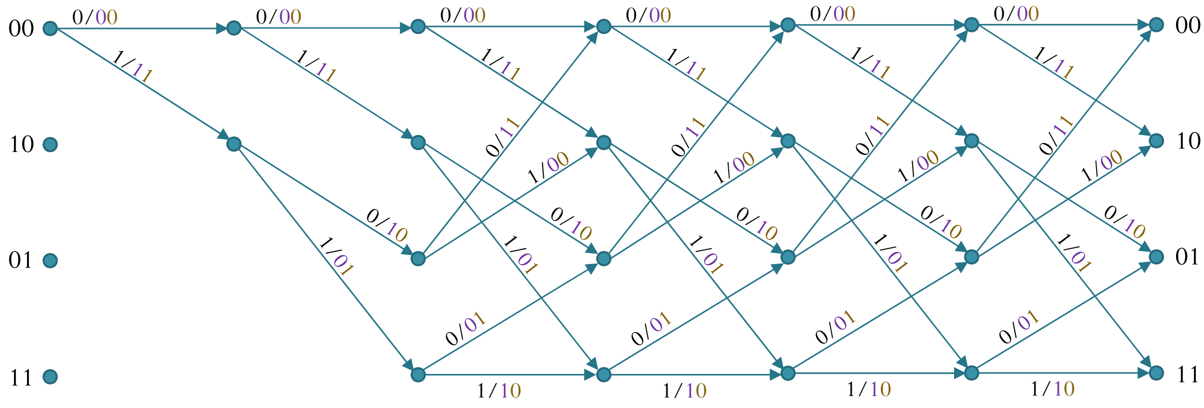

Figure 8.2: State diagram for a convolutional encoder

Problem 2. Consider a convolutional encoder whose trellis diagram is given in Figure 8.2.

(a) Find the code rate

(b) Suppose the data bits (message) are $\underline{\mathbf{b}} = [0100101]$. Find the corresponding codeword $\underline{\mathbf{x}}$.

(c) Find the data vector $\underline{\mathbf{b}}$ which gives the codeword $\underline{\mathbf{x}} = [001110111110]$.

(d) Suppose that we observe $\underline{\mathbf{y}} = [001110000101]$ at the input of the minimum distance decoder. Explain why we can easily find the decoded codeword $\hat{\underline{\mathbf{x}}}$ and the decoded message $\hat{\underline{\mathbf{b}}}$ without applying the Viterbi algorithm.

(e) Suppose that we observe $\underline{\mathbf{y}} = [010101111110]$ at the input of the minimum distance decoder. Use Viterbi algorithm to find the decoded codeword $\hat{\underline{\mathbf{x}}}$ and the decoded message $\hat{\underline{\mathbf{b}}}$. Show your work on Figure 8.3 below.

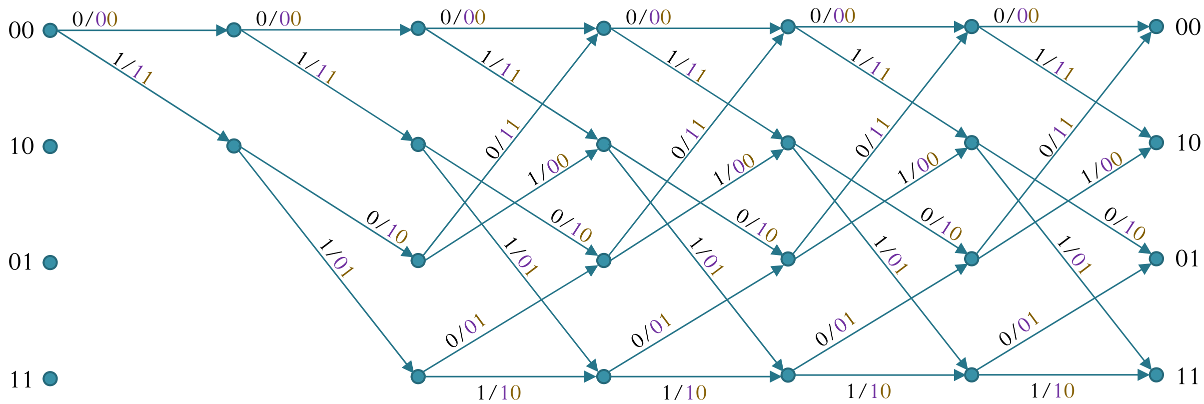

Figure 8.3: State diagram for a convolutional encoder

Make sure that all the running (cumulative) path metric are shown and the discarded branches are indicated at every steps.

Problem 3. Consider four vectors:

$$\mathbf{v}^{(1)} = \begin{pmatrix} +1 \\ -1 \\ +1 \\ 0 \\ -1 \end{pmatrix}, \mathbf{v}^{(2)} = \begin{pmatrix} +1 \\ +1 \\ 0 \\ +1 \\ 0 \end{pmatrix}, \mathbf{v}^{(3)} = \begin{pmatrix} +2 \\ 0 \\ +1 \\ +1 \\ -1 \end{pmatrix}, \text{ and } \mathbf{v}^{(4)} = \begin{pmatrix} +3 \\ +1 \\ +1 \\ +2 \\ -1 \end{pmatrix}.$$

(a) Use the Gram-Schmidt orthogonalization procedure (GSOP) (where the vectors are applied <u>**in the order given**</u>) to find the orthonormal vectors $\mathbf{e}^{(1)}, \mathbf{e}^{(2)}, \ldots$ that can be used as axes to represent $\mathbf{v}^{(1)}, \mathbf{v}^{(2)}, \mathbf{v}^{(3)}$, and $\mathbf{v}^{(4)}$.

(b) Find the corresponding vectors $\mathbf{c}^{(1)}, \mathbf{c}^{(2)}, \mathbf{c}^{(3)}$, and $\mathbf{c}^{(4)}$ that represent $\mathbf{v}^{(1)}, \mathbf{v}^{(2)}, \mathbf{v}^{(3)}$, and $\mathbf{v}^{(4)}$ in the new axes derived in the previous part.