| **ECS 452: Digital Communication Systems** | **2016/2** |
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# HW 7 — Due: May 2

*Lecturer: Asst. Prof. Dr. Prapun Suksompong*

**Instructions**

(a) This assignment has 4 pages.

(b) (1 pt) Write your first name and the last three digit of your student ID on the upper-right corner of ***every*** submitted sheet.

(c) (1 pt) For each part, write your explanation/derivation and answer in the space provided.

(d) (8 pt) It is important that you try to solve all non-optional problems.

(e) Late submission will be heavily penalized.

**Problem 1.** Continue from the previous assignment. Consider a block code whose generator matrix is

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

(a) Suppose we receive $\underline{\mathbf{y}} = [1\,1\,1\,1\,0\,1]$.

(i) Minimum distance decoding:

i. Find the distance $d(\underline{\mathbf{x}}, \underline{\mathbf{y}})$ between this received vector $\underline{\mathbf{y}}$ and each of the possible codewords $\underline{\mathbf{x}}$.

| $\underline{\mathbf{b}}$ | | | $\underline{\mathbf{x}}$ | | | | | | $d(\underline{\mathbf{x}}, \underline{\mathbf{y}})$ |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 0 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | |
| 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | |
| 0 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | |
| 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | |
| 1 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | |
| 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | |
| 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | |

ii. Use the answer in the previous part to find $\underline{\hat{\mathbf{x}}}$ and $\underline{\hat{\mathbf{b}}}$

(ii) Decoding via the syndrome:

i. Find the parity check matrix $\mathbf{H}$ of this code.

ii. Find the syndrome vector $\underline{\mathbf{s}}$.

iii. Use the answer in the previous parts to find $\underline{\hat{\mathbf{x}}}$ and $\underline{\hat{\mathbf{b}}}$

**Problem 2.** In the previous assignment, we consider the following encoding for a systematic linear block code:

- The bit positions that are powers of 2 (1, 2, 4, 8, 16, etc.) are check bits.
- The rest (3, 5, 6, 7, 9, etc.) are filled up with the $k$ data bits.
- Each check bit forces the parity of some collection of bits, including itself, to be even.
  - To see which check bits the data bit in position $i$ contributes to, rewrite $i$ as a sum of powers of 2. A bit is checked by just those check bits occurring in its expansion.

For the case when the codeword's length $n = 7$, we found that

$$\mathbf{H} = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

(a) Explain, from the elements inside the matrix $\mathbf{H}$, how this is a Hamming code.

(b) Consider the following decoding instruction:

- When a vector is observed, the receiver initializes a counter to zero. It then examines each check bit at position $i$ ($i = 1, 2, 4, 8, ...$) to see if it gives the correct parity.
- If not, the receiver adds $i$ to the counter. If the counter is zero after all the check bits have been examined (i.e., if they were all correct), the observed vector is accepted as a valid codeword. If the counter is nonzero, it contains the position of the incorrect bit.

Explain how the instruction above is the "same" as the decoding via the syndrome described in class.

**Problem 3.** Consider a convolutional encoder whose state diagram is given in Figure 7.1.

(a) Find the code rate

(b) Suppose the data bits (message) are 0100101. Find the corresponding codeword.

(c) Find the data vector $\underline{\mathbf{b}}$ which gives the codeword $\underline{\mathbf{x}} = [001110111110110011]$

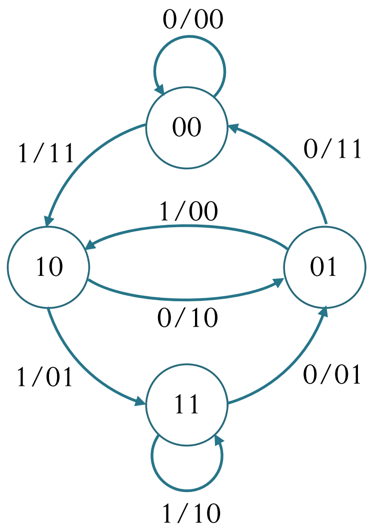


Figure 7.1: State diagram for a convolutional encoder

**Problem 4.** Construct a generator matrix $\mathbf{G}$ and a corresponding parity check matrix $\mathbf{H}$ for a (15,11) Hamming code.