ECS 452: Digital Communication Systems	**2016/2**

HW 6 — Due: April 25

Lecturer: Asst. Prof. Dr. Prapun Suksompong

Instructions

(a) This assignment has 6 pages.

(b) (1 pt) Write your first name and the last three digit of your student ID on the upper-right corner of ***every*** submitted sheet.

(c) (1 pt) For each part, write your explanation/derivation and answer in the space provided.

(d) (8 pt) It is important that you try to solve all non-optional problems.

(e) Late submission will be heavily penalized.

Problem 1. Consider a single-parity-check linear code. For each of the data block below, find the corresponding codeword.

$\underline{\mathbf{b}}$	$\underline{\mathbf{x}}$
010	
111	
001	

Problem 2. For each of the codes below, check whether it is a linear code.

(a) $\mathcal{C} = \{000, 001, 100, 101\}$

(b) $\mathcal{C} = \{000, 100, 110, 111\}$

(c) $\mathcal{C} = \{001, 100, 101\}$

Problem 3. Consider a block code whose generator matrix is

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

(a) Find its code rate.

(b) Suppose the message is $\underline{\mathbf{b}} = [1\,0\,1]$. Find the corresponding codeword $\underline{\mathbf{x}}$.

(c) In the provided table, list all possible data (message) vectors $\underline{\mathbf{b}}$ in the left column (one in each row). Then, find the corresponding codewords $\underline{\mathbf{x}}$ and their weights in the second and third columns, respectively.

$\underline{\mathbf{b}}$	$\underline{\mathbf{x}}$	$w(\underline{\mathbf{x}})$

(d) Find the minimum distance $d_{\min}$ for this code.

Problem 4. Consider a block code whose generator matrix is

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Let the row vector $\underline{\mathbf{g}}^{(i)}$ represents the ith row of $\mathbf{G}$. Observe that $\underline{\mathbf{g}}^{(3)} = \underline{\mathbf{g}}^{(1)} + \underline{\mathbf{g}}^{(2)}$. Why is this bad?

Problem 5. Consider the following encoding for a systematic linear block code:

- The bit positions that are powers of 2 (1, 2, 4, 8, 16, etc.) are check bits.
- The rest (3, 5, 6, 7, 9, etc.) are filled up with the k data bits.
- Each check bit forces the parity of some collection of bits, including itself, to be even.
  - To see which check bits the data bit in position i contributes to, rewrite i as a sum of powers of 2. A bit is checked by just those check bits occurring in its expansion.

We will consider the case when the codeword's length $n = 7$.

(a) How many bits are check bits?
Hint: How many bit positions are powers of 2?

(b) Find the generator matrix $\mathbf{G}$ for this code.

(c) Find the corresponding parity check matrix **H**.

Problem 6. Consider each of the block codes whose codebooks are provided below. For each code, is the code linear? If so, find the corresponding generator matrix. If not, find a pair of codewords whose sum is not a codeword.

(a)

$\underline{\mathbf{b}}$			$\underline{\mathbf{c}}$				
0	0	0	0	0	0	0	0
0	0	1	0	0	1	1	0
0	1	0	1	0	1	0	1
0	1	1	1	0	0	1	1
1	0	0	1	1	1	0	0
1	0	1	1	1	0	1	0
1	1	0	0	1	0	0	1
1	1	1	0	1	1	1	1

(b)

$\underline{\mathbf{b}}$			$\underline{\mathbf{c}}$				
0	0	0	0	0	0	0	0
0	0	1	0	0	1	1	0
0	1	0	1	0	1	0	1
0	1	1	1	0	0	1	1
1	0	0	1	1	1	0	0
1	0	1	1	1	1	1	0
1	1	0	0	1	0	0	1
1	1	1	0	1	1	1	1

Extra Questions

Here are some optional questions for those who want more practice.

Problem 7 (Carlson and Crilly, 2009, P13.2-1)**.** In mathematical analysis, a function $d(\underline{\mathbf{x}}, \underline{\mathbf{y}})$ is a "true" distance if it satisfies all of the following properties:

(i) positivity: $d(\underline{\mathbf{x}}, \underline{\mathbf{y}}) \geq 0$ with equality if and only if $\underline{\mathbf{x}} = \underline{\mathbf{y}}$

(ii) symmetry: $d(\underline{\mathbf{x}}, \underline{\mathbf{y}}) = d(\underline{\mathbf{y}}, \underline{\mathbf{x}})$

(iii) triangle inequality: $d(\underline{\mathbf{x}}, \underline{\mathbf{z}}) \leq d(\underline{\mathbf{x}}, \underline{\mathbf{y}}) + d(\underline{\mathbf{y}}, \underline{\mathbf{z}})$

Is the Hamming distance a "true" distance? (Prove or disprove)

Hint: For the triangle inequality, first consider the number of 1s in $\underline{\mathbf{u}}$, $\underline{\mathbf{v}}$, and $\underline{\mathbf{u}} \bigoplus \underline{\mathbf{v}}$ and confirm that $d(\underline{\mathbf{u}}, \underline{\mathbf{v}}) \leq w(\underline{\mathbf{u}}) + w(\underline{\mathbf{v}})$. Then, from this inequality, replace $\underline{\mathbf{u}}$ by $\underline{\mathbf{x}} \bigoplus \underline{\mathbf{y}}$ and $\underline{\mathbf{v}}$ by $\underline{\mathbf{y}} \bigoplus \underline{\mathbf{z}}$.

Problem 8 (Carlson and Crilly, 2009, P13.2-2 and P13.2-3)**.** Consider a block code. Suppose $\underline{\mathbf{x}}$ is the transmitted codeword and that $\underline{\mathbf{y}}$ is the vector that results when $\underline{\mathbf{x}}$ is received with $i > 0$ bit errors. Use the triangle inequality for the Hamming distance to show that

(a) if the code has $d_{\min} \geq \ell + 1$ and if $i \leq \ell$, then the errors are detectable.

(b) if the code has $d_{\min} \geq 2t + 1$ and if $i \leq t$, then the errors are correctable by the minimum distance decoder.