ECS 452: Digital Communication Systems	**2016/2**

HW 4 — Due: March 14

Lecturer: Asst. Prof. Dr. Prapun Suksompong

Instructions

(a) This assignment has 6 pages.

(b) (1 pt) Write your first name and the last three digit of your student ID on the upper-right corner of ***every*** submitted sheet.

(c) (1 pt) For each part, write your explanation/derivation and answer in the space provided.

(d) (8 pt) It is important that you try to solve all non-optional problems.

(e) Late submission will be heavily penalized.

Problem 1. Consider a repetition code with a code rate of 1/5. Assume that the code is used with a BSC with a crossover probability $p = 0.4$. Find the ML detector and its error probability.

Problem 2. A channel encoder map blocks of two bits to five-bit (channel) codewords. The four possible codewords are 00000, 01000, 10001, and 11111. A codeword is transmitted over the BSC with crossover probability $p = 0.1$.

(a) What is the minimum (Hamming) distance d_{min} among the codewords?

(b) Suppose the codeword $\underline{\mathbf{x}} = 10001$ was transmitted. What is the probability that the receiver observes $\underline{\mathbf{y}} = 01001$ at the output of the BSC.

(c) Assume that all four codewords are equally likely to be transmitted. Suppose the receiver observes 01001 at the output of the BSC. What is the most likely codeword that was transmitted?

Problem 3. Consider random variables X and Y whose joint pmf is given by

$$p_{X,Y}(x,y) = \begin{cases} c\,(x+y)\,, & x \in \{1,3\} \text{ and } y \in \{2,4\}\,, \\ 0, & \text{otherwise.} \end{cases}$$

Evaluate the following quantities.

(a) c

(b) $H(X,Y)$

(c) $H(X)$

(d) $H(Y)$

(e) $H(X|Y)$

(f) $H(Y|X)$

(g) $I(X;Y)$

Problem 4. Consider a pair of random variables X and Y whose joint pmf is given by

$$p_{X,Y}(x,y) = \begin{cases} 1/15, & x=3, y=1, \\ 2/15, & x=4, y=1, \\ 4/15, & x=3, y=3, \\ \beta, & x=4, y=3, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the value of the constant β.

(b) Are X and Y independent?

(c) Evaluate the following quantities.

(i) $H(X)$

(ii) $H(Y)$

(iii) $H(X,Y)$

(iv) $H(X|Y)$

(v) $H(Y|X)$

(vi) $I(X;Y)$

Extra Questions

Here are some optional questions for those who want more practice.

Problem 5. Consider a repetition code with a code rate of 1/5. Assume that the code is used with a BSC with a crossover probability $p = 0.4$.

(a) Suppose the info-bit S is generated with

$$P[S = 0] = 1 - P[S = 1] = 0.4.$$

Find the MAP detector and its error probability.

(b) Assume the info-bit S is generated with

$$P[S=0] = 1 - P[S=1] = 0.45.$$

Suppose the receiver observes 01001.

(i) What is the probability that 0 was transmitted? (Do not forget that this is a conditional probability. The answer is not 0.45 because we have some extra information from the observed bits at the receiver.)

(ii) What is the probability that 1 was transmitted?

(iii) Given the observed 01001, which event is more likely, $S=1$ was transmitted or $S=0$ was transmitted? Does your answer agree with the majority voting rule for decoding?

(c) Assume that the source produces source bit S with

$$P[S=0] = 1 - P[S=1] = p_0.$$

Suppose the receiver observes 01001.

(i) What is the probability that 0 was transmitted?

(ii) What is the probability that 1 was transmitted?

(iii) Given the observed 01001, which event is more likely, $S = 1$ was transmitted or $S = 0$ was transmitted? Your answer may depend on the value of p_0. Does your answer agree with the majority voting rule for decoding?

Problem 6. A channel encoder map blocks of two bits to five-bit (channel) codewords. The four possible codewords are 00000, 01000, 10001, and 11111. Assume that the four codewords are not equally likely. Suppose 11111 is transmitted more frequently with probability 0.7. The other three codewords are transmitted with probability 0.1 each.

A codeword was transmitted over the BSC with crossover probability $p = 0.1$. Suppose the receiver observes 01001 at the output of the BSC. What is the most likely codeword that was transmitted?