HW 3 — Due: March 2

Lecturer: Asst. Prof. Dr. Prapun Suksompong

Instructions

(a) This assignment has 5 pages.

(b) (1 pt) Write your first name and the last three digit of your student ID on the upper-right corner of **every** submitted sheet.

(c) (1 pt) For each part, write your explanation/derivation and answer in the space provided.

(d) (8 pt) It is important that you try to solve all non-optional problems.

(e) Late submission will be heavily penalized.

Problem 1. A DMC has $\mathcal{X} = \{0, 1\}$ and $\mathcal{Y} = \{1, 2, 3\}$. The following decoding table is used to decode the channel output.

y	$\hat{x}(y)$
1	0
2	1
3	0

Suppose the channel output string is 212213221133122122132.

(a) Find the corresponding decoded string.

(b) Suppose the channel input string is produced from an ASCII source encoder by the command `dec2bin(SourceString,7)` in MATLAB. Assume that there is no channel decoding error. Find the corresponding source string.

Problem 2. Consider a BSC whose crossover probability for each bit is $p = 0.35$. Suppose $P[X = 0] = 0.45$.

(a) Analyze the performance of all four reasonable detectors for this binary channel. Complete the table below:

$\hat{x}(y)$	$\hat{x}(0)$	$\hat{x}(1)$	$P(\mathcal{C})$	$P(\mathcal{E})$
y				
$1-y$				
1				
0				

(b) Find the MAP detector and its error probability.

(c) Find the ML detector and its error probability.

Problem 3. Consider a DMC whose $\mathcal{X} = \{1, 2, 3\}$, $\mathcal{Y} = \{1, 2, 3\}$, and $\mathbf{Q} = \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.2 & 0.6 \end{bmatrix}$.

Suppose the input probability vector is $\underline{\mathbf{p}} = [0.2, 0.4, 0.4]$.

(a) Find the error probability of the naive decoder.

(b) Find the error probability of the (DIY) decoder $\hat{x}(y) = 4 - y$.

(c) Find the MAP detector and its error probability.

(d) Find the ML detector and its error probability.

Problem 4. Consider a BAC whose $Q(1|0) = 0.35$ and $Q(0|1) = 0.55$. Suppose $P[X = 0] = 0.4$.

(a) Analyze the performance of all four reasonable detectors for this binary channel. Complete the table below:

$\hat{x}(y)$	$\hat{x}(0)$	$\hat{x}(1)$	$P(\mathcal{C})$	$P(\mathcal{E})$
y				
$1 - y$				
1				
0				

(b) Find the MAP detector and its error probability.

(c) Find the ML detector and its error probability.

Extra Question

Here is an optional question for those who want more practice.

Problem 5. Consider a DMC whose samples of input X and output Y are recorded as row vectors $\underline{\mathbf{x}}$ and $\underline{\mathbf{y}}$ in the file HW_DMC_Channel_Data.mat. Write MATLAB script which uses the recorded information to estimate the quantities below. Note that most of these can be solved by applying appropriate parts of the codes shown in class.

(a) The support $\mathcal{X}$ of X.

(b) The support $\mathcal{Y}$ of Y.

(c) The row vector $\mathbf{p}$ which contains the pmf of X.

(d) The $\mathbf{Q}$ matrix.

(e) The row vector $\mathbf{q}$ which contains the pmf of Y. Do this using two methods:

  (i) Count directly from the observed values of Y.

  (ii) Use the estimated values of $\underline{\mathbf{p}}$ and $\mathbf{Q}$.

(f) The error probability when the naive decoder is used. Do this using two methods:

  (i) Directly construct $\underline{\hat{\mathbf{x}}}$ from $\underline{\mathbf{y}}$. Then, compare $\underline{\hat{\mathbf{x}}}$ and $\underline{\mathbf{x}}$.

  (ii) Use the estimated values of $\underline{\mathbf{p}}$ and $\mathbf{Q}$.

(g) The error probability when the MAP decoder is used. Do this using two methods:

  (i) First find the MAP decoder table using the estimated values of $\underline{\mathbf{p}}$ and $\mathbf{Q}$. Then, construct $\underline{\hat{\mathbf{x}}}$ from $\underline{\mathbf{y}}$ according to the decoder table. Finally, compare $\underline{\hat{\mathbf{x}}}$ and $\underline{\mathbf{x}}$.

  (ii) Use the estimated values of $\underline{\mathbf{p}}$ and $\mathbf{Q}$ to directly calculate the error probability.

(h) The error probability when the ML decoder is used. Do this using two methods:

  (i) First find the ML decoder table using the estimated value of $\mathbf{Q}$. Then, construct $\underline{\hat{\mathbf{x}}}$ from $\underline{\mathbf{y}}$ according to the decoder table. Finally, compare $\underline{\hat{\mathbf{x}}}$ and $\underline{\mathbf{x}}$.

  (ii) Use the estimated values of $\underline{\mathbf{p}}$ and $\mathbf{Q}$ to directly calculate the error probability.