ECS 452: Digital Communication Systems **2016/2**

HW 1 — Due: Feb 9

Lecturer: Asst. Prof. Dr. Prapun Suksompong

Instructions

(a) This assignment has 4 pages.

(b) (1 pt) Write your first name and the last three digit of your student ID on the upper-right corner of ***every*** submitted sheet.

(c) (1 pt) For each part, write your explanation/derivation and answer in the space provided.

(d) (8 pt) It is important that you try to solve all non-optional problems.

(e) Late submission will be heavily penalized.

Problem 1. Consider the code $\{0, 01\}$

(a) Is it nonsingular?

(b) Is it uniquely decodable?

(c) Is it prefix-free?

Problem 2. In this question, each output string from a DMS is encoded by the following source code:

x	Codeword $c(x)$
'a'	1
'd'	01
'e'	0000
'i'	001
'o'	00010
'u'	00011

(a) Is the code prefix-free?

(b) Is the code uniquely decodable?

(c) Suppose the DMS produces the string 'audio'. Find the output of the source encoder.

(d) Suppose the output of the source encoder is
0 0 0 1 0 0 1 0 0 0 0 0 0 1 0 1 0 0 0 0 1 0 1 0 0 1 0 0 0 0

Find the corresponding source string produced by the DMS. Use "/" to indicate the locations where the sting above is split into codewords.

Problem 3. Consider the random variable X whose support S_X contains seven values:

$$S_X = \{x_1, x_2, \ldots, x_7\}.$$

Their corresponding probabilities are given by

x	x_1	x_2	x_3	x_4	x_5	x_6	x_7
$p_X(x)$	0.49	0.26	0.12	0.04	0.04	0.03	0.02

(a) Find the entropy $H(X)$.

(b) Find a binary Huffman code for X.

(c) Find the expected codelength for the encoding in part (b).

Problem 4. Find the entropy and the binary Huffman code for the random variable X with pmf

$$p_X(x) = \begin{cases} \frac{x}{21}, & x = 1, 2, \ldots, 6, \\ 0, & \text{otherwise.} \end{cases}$$

Also calculate $\mathbb{E}[\ell(X)]$ when Huffman code is used.

Problem 5. These codes cannot be Huffman codes. Why?

(a) $\{00, 01, 10, 110\}$

(b) $\{01, 10\}$

(c) $\{0, 01\}$

Extra Questions

Here are some optional questions for those who want more practice.

Problem 6. (Optional) The following claim is sometimes found in the literature:

> "It can be shown that the length $\ell(x)$ of the Huffman code of a symbol x with probability $p_X(x)$ is always less than or equal to $\lceil -\log_2 p_X(x) \rceil$".

Even though it is correct in many cases, this claim is not true in general.

Find an example where the length $\ell(x)$ of the Huffman code of a symbol x is greater than $\lceil -\log_2 p_X(x) \rceil$.

Hint: Consider a pmf that has the following four probability values $\{0.01, 0.30, 0.34, 0.35\}$.

Problem 7. (Optional) Construct a random variable X (by specifying its pmf) whose corresponding Huffman code is $\{0, 10, 11\}$.