

# ECS 452: In-Class Exercise # 9

## Instructions

1. Separate into groups of no more than three persons.
2. The group cannot be the same as your former group.
3. Only one submission is needed for each group.
4. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
5. **Do not panic.**

Date: <b>28/03/2017</b>			
Name			ID (last 3 digits)
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1. Assume GF(2). Calculate the following quantities:

a.  $1 \oplus 0 = 1$

b.  $0 \oplus 1 \oplus 1 = 0$

c.  $1 \cdot 0 = 0$

d.  $0 \cdot 1 \cdot 1 = 0$

e.  $[0 \ 1 \ 1] \oplus [1 \ 0 \ 0] = [1 \ 1 \ 1]$

f.  $[0 \ 1 \ 1] \oplus [0 \ 1 \ 1] = [0 \ 0 \ 0]$

g.  $[0 \ 1 \ 1] \cdot \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 0 \end{bmatrix} = [0 \ 1]$

$0 \cdot 1 \oplus 1 \cdot 1 \oplus 1 \cdot 1 = 0 \oplus 1 \oplus 1 = 0$   
 $0 \cdot 0 \oplus 1 \cdot 1 \oplus 1 \cdot 0 = 0 \oplus 1 \oplus 0 = 1$

2. A codeword  $\underline{x} = [1 \ 0 \ 1 \ 1]$  is sent over the BSC. Suppose the error pattern is  $\underline{e} = [0 \ 0 \ 1 \ 1]$ . Find the observed vector at the receiver.

$$\underline{y} = \underline{x} \oplus \underline{e} = [1 \ 0 \ 1 \ 1] \oplus [0 \ 0 \ 1 \ 1] = [1 \ 0 \ 0 \ 0]$$

3. A codeword  $[1 \ 0 \ 1 \ 1]$  is sent over the BSC. Suppose the observed vector at the receiver is  $\underline{y} = [0 \ 1 \ 1 \ 1]$ . Find the error pattern.

$$\underline{y} = \underline{x} \oplus \underline{e} \Rightarrow \underline{e} = \underline{x} \oplus \underline{y} = [1 \ 0 \ 1 \ 1] \oplus [0 \ 1 \ 1 \ 1] = [1 \ 1 \ 0 \ 0]$$

Alternatively, simply observe that these correspond to the positions that  $\underline{x}$  and  $\underline{y}$  are different.

4. A codeword is sent over the BSC.

Suppose the observed vector at the receiver is  $\underline{y} = [0 \ 1 \ 1 \ 1]$  and the error pattern is  $\underline{e} = [0 \ 0 \ 1 \ 1]$ .

Find the transmitted codeword.

$$\underline{y} = \underline{x} \oplus \underline{e} \Rightarrow \underline{x} = \underline{y} \oplus \underline{e} = [0 \ 1 \ 1 \ 1] \oplus [0 \ 0 \ 1 \ 1] = [0 \ 1 \ 0 \ 0]$$

Alternatively, the error pattern says that the last two bits in the observed vector are received incorrectly. So, we need to flip their values to get the transmitted codeword.