ECS 452: In-Class Exercise # 8

Instructions

1. Separate into groups of no more than three persons.
2. The group cannot be the same as your former group.
3. Only one submission is needed for each group.
4. **Do not panic.**

Date: 14/03/2017			
Name	ID (last 3 digits)		
Prapun	5	5	5

1. For each of the following DMC's probability transition matrices **Q**, (i) indicate whether the corresponding DMC is weakly symmetric (Yes or No), (ii) evaluate the corresponding capacity value (your answer should be of the form X.XXXX), and (iii) specify the channel input pmf (a row vector **p**) that achieves the capacity.

Check that
① all the rows of Q are permutations of each other
② all the column sums are equal

Q	Weakly Symmetric?	C	**p**
crossover probability p $\begin{bmatrix} 1/3 & 2/3 \\ 2/3 & 1/3 \end{bmatrix}$ This is the Q matrix for a BSC.	Yes. BSC is symmetric and hence weakly symmetric.	For BSC, $C = 1 - h(p)$ $= 1 - h(2/3)$ $\approx 1 - 0.9183$ ≈ 0.0817.	C is achieved by uniform X on $\mathcal{X}$ $p^* = [\frac{1}{2} \ \frac{1}{2}]$
r $\begin{bmatrix} 1/6 & 1/3 & 1/2 \\ 1/2 & 1/6 & 1/3 \\ 1/3 & 1/2 & 1/6 \end{bmatrix}$	① ✓ ② ✓ Yes	$\log_2\lvert\mathcal{Y}\rvert - H(r)$ $= \log_2\lvert 3\rvert - H([\frac{1}{2} \ \frac{1}{3} \ \frac{1}{6}])$ ≈ 0.1258. [bpcu]	C is achieved by uniform X on $\mathcal{X}$ $p^* = [\frac{1}{3} \ \frac{1}{3} \ \frac{1}{3}]$
$\begin{bmatrix} 0 & 1/2 & 0 & 1/2 & 0 & 0 \\ 1/3 & 0 & 1/3 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$	① × ② × No	Note that there is only one non-zero element in each column ⇒ This is NO^2 channel ⇒ $C = \log_2\lvert\mathcal{X}\rvert = \log_2 3 \approx 1.5850$.	⇒ C is achieved by uniform X $p^* = [\frac{1}{3} \ \frac{1}{3} \ \frac{1}{3}]$
$\begin{bmatrix} 2/3 & 1/6 & 1/6 \\ 2/3 & 1/6 & 1/6 \end{bmatrix}$	① ✓ ② × No	Note that all the rows of Q are the same ⇒ $Q(y\lvert x)$ does not depend on x ⇒ $X \perp Y$ ⇒ $I(X;Y) = 0$ for any $p(x)$ ⇒ $C = 0.0000$ [bpcu]	Any p^* will give the same $I(X;Y) = C = 0$.

In this problem, $\lvert\mathcal{X}\rvert = 2$. Therefore, p can be any row vector of length = 2

$[p_1 \ p_2]$

that represents a pmf which means

$p_1, p_2 \geq 0$

$p_1 + p_2 = 0$