ECS 452: In-Class Exercise # 6

Instructions

Name	ID (last 3 digits)		
Prapun	**5**	**5**	**5**

1. Separate into groups of no more than three persons.
2. The group cannot be the same as your former group.
3. Only one submission is needed for each group.
4. *Write down all the steps* that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
5. **Do not panic.**

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1. Consider two random variables X and Y whose joint pmf matrix is given by

$$P = \begin{array}{c|cc} {}_x\backslash^y & 3 & 5 \\ \hline 1 & 1/18 & 5/18 \\ 2 & 4/9 & 2/9 \end{array}$$

$\xrightarrow{\Sigma} \frac{6}{18} = \frac{1}{3}$ = $p(x)$

$\xrightarrow{\Sigma} \frac{6}{9} = \frac{2}{3}$

$g(y)$: $\frac{9}{18}$ $\frac{9}{18}$

Calculate the following quantities. Except for part (e), all of your answers should be of the form X.XXXX.

a. $H(X,Y) = -\frac{1}{18}\log_2\frac{1}{18} - \frac{5}{18}\log_2\frac{5}{18} - \frac{4}{9}\log_2\frac{4}{9} - \frac{2}{9}\log_2\frac{2}{9}$

≈ 1.7472 bits (per symbol)

$* \frac{1}{3}\times0.65 + \frac{2}{3}\times0.9183 \approx 0.8289$

b. $H(X) = H\left(\left[\begin{array}{cc}1/3 & 2/3\end{array}\right]\right) = -\frac{1}{3}\log_2\frac{1}{3} - \frac{2}{3}\log_2\frac{2}{3} \approx 0.9183$ bits (per symbol)

c. $H(Y) = H\left(\left[\begin{array}{cc}1/2 & 1/2\end{array}\right]\right) = \log_2 2 = 1.000$ bit (per symbol)

$\quad$ For uniform RV, the entropy is simply $\log_2$ of the size of its support.

d. $H(Y|X) = H(X,Y) - H(X) \approx 1.7472 - 0.9183$

≈ 0.8289 bits (per symbol)

Alternatively,

$H(Y|X)$
$= p(1)H(Y|1) + p(2)H(Y|2)$
$* \frac{1}{3}\times0.65 + \frac{2}{3}\times0.9183 \approx 0.8289$

$H(Y|1) = H\left(\left[\begin{array}{cc}1/6 & 5/6\end{array}\right]\right) \approx 0.6500$ bits (per symbol)

e. **Q** matrix

$P = \begin{bmatrix} 1/18 & 5/18 \\ 4/9 & 2/9 \end{bmatrix}$

$\xrightarrow{\times \frac{1}{p(x)}}$ $\xrightarrow{\times 3}$ $\xrightarrow{\times 3/2}$

$\begin{array}{c|cc} {}_x\backslash^y & 3 & 5 \\ \hline 1 & 1/6 & 5/6 \\ 2 & 2/3 & 1/3 \end{array} = Q$

$H(Y|2) = H\left(\left[\begin{array}{cc}2/3 & 1/3\end{array}\right]\right) \approx 0.9183$ bits (per symbol)