Solution ECS 452: In-Class Exercise # 5

Instructions

1. Separate into groups of no more than three persons.
2. The group cannot be the same as your former group.
3. Only one submission is needed for each group.
4. ***Write down all the steps*** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
5. **Do not panic.**

Date: 16/ 02/ 2017			
Name	ID (last 3 digits)		
Prapun	5	5	5

1. Consider a DMC whose $\mathcal{X}=\{1,2,3\}$, $\mathcal{Y}=\{1,2,3,4\}$, and $\mathbf{Q}=\begin{bmatrix} 0.2 & 0.6 & 0.1 & 0.1 \\ 0.1 & 0.7 & 0.1 & 0.1 \\ 0.3 & 0.3 & 0.3 & 0.1 \end{bmatrix}$.

   Suppose the input probability vector is $\underline{\mathbf{p}}=[0.2 \quad 0.1 \quad 0.7]$.

   a. Suppose the following decoder is used. Find the corresponding $P(\mathcal{E})$.

y	$\hat{x}(y)$
1	1
2	1
3	2
4	3

$\hat{x}(y)$ 1 1 2 3

$P = \begin{array}{c|cccc} x\backslash y & 1 & 2 & 3 & 4 \\ 1 & 0.04 & 0.12 & 0.02 & 0.02 \\ 2 & 0.01 & 0.07 & 0.01 & 0.01 \\ 3 & 0.21 & 0.21 & 0.21 & 0.07 \end{array}$

$P(C) = 0.04 + 0.12 + 0.01 + 0.07$

$= 0.24$

$P(\mathcal{E}) = 1 - 0.24 = 0.76$

even worse than the naive decoder.

   b. Find the MAP detector and its error probability.

$P = \begin{array}{c|cccc} x\backslash y & 1 & 2 & 3 & 4 \\ 1 & 0.04 & 0.12 & 0.02 & 0.02 \\ 2 & 0.01 & 0.07 & 0.01 & 0.01 \\ 3 & 0.21 & 0.21 & 0.21 & 0.07 \end{array}$

For each column of the P matrix, select the max value.

The corresponding x-value for the selected value in each column.

y	$\hat{x}_{MAP}(y)$
1	3
2	3
3	3
4	3

So, $\hat{x}_{MAP}(y) \equiv 3$.

$P(C) = 0.21 + 0.21 + 0.21 + 0.07 = 0.7$

$P(\mathcal{E}) = 1 - P(C) = 1 - 0.7 = 0.3$

   c. Find the ML detector and its error probability.

For each column of the Q matrix, select the max value.

$Q = \begin{array}{c|cccc} x\backslash y & 1 & 2 & 3 & 4 \\ 1 & 0.2 & 0.6 & 0.1 & 0.1 \\ 2 & 0.1 & 0.7 & 0.1 & 0.1 \\ 3 & 0.3 & 0.3 & 0.3 & 0.1 \end{array}$ $\times 0.2$, $\times 0.1$, $\times 0.7$

$= P = \begin{array}{c|cccc} x\backslash y & 1 & 2 & 3 & 4 \\ 1 & 0.04 & 0.12 & 0.02 & 0.02 \\ 2 & 0.01 & 0.07 & 0.01 & 0.01 \\ 3 & 0.21 & 0.21 & 0.21 & 0.07 \end{array}$

Select the same elements as in the Q matrix.

The corresponding x-value for the selected value in each column

y	$\hat{x}_{ML}(y)$
1	3
2	2
3	3
4	1

All values in the last column are the same. So, we can use any of them.

$P(C) = 0.21 + 0.07 + 0.21 + 0.02 = 0.51$

$P(\mathcal{E}) = 1 - 0.51 = 0.49$

Alternative answer

y	$\hat{x}_{ML}(y)$
1	3
2	2
3	3
4	2

$P(C) = 0.21 + 0.07 + 0.21 + 0.01 = 0.50$

$P(\mathcal{E}) = 1 - 0.5 = 0.5$

Another alternative answer

y	$\hat{x}_{ML}(y)$
1	3
2	2
3	3
4	3

$P(C) = 0.21 + 0.07 + 0.21 + 0.07 = 0.56$

$P(\mathcal{E}) = 1 - 0.56 = 0.44$