

Instructions

1. Separate into groups of no more than three persons.
2. The group cannot be the same as your former group.
3. Only one submission is needed for each group.
4. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
5. **Do not panic.**

Date: 14/02/2017			
Name			ID (last 3 digits)
Prapun			5 5 5

1. Consider a DMC whose $\mathcal{X} = \{1, 2, 3\}$, $\mathcal{Y} = \{1, 2, 3, 4\}$, and $\mathbf{Q} = \begin{bmatrix} 0.2 & 0.6 & 0.1 & 0.1 \\ 0.1 & 0.7 & 0.1 & 0.1 \\ 0.3 & 0.3 & 0.3 & 0.1 \end{bmatrix}$.

Suppose the input probability vector is $\underline{p} = [0.2 \ 0.1 \ 0.7]$.

a. Find the output probability vector \underline{q}

$$\underline{q} = \underline{p} \mathbf{Q} = [0.2 \ 0.1 \ 0.7] \begin{bmatrix} 0.2 & 0.6 & 0.1 & 0.1 \\ 0.1 & 0.7 & 0.1 & 0.1 \\ 0.3 & 0.3 & 0.3 & 0.1 \end{bmatrix} = [0.26 \ 0.4 \ 0.24 \ 0.1]$$

b. Find the joint pmf matrix \mathbf{P} .

Multiply each row in the \mathbf{Q} matrix by its corresponding $p(x)$

$$\mathbf{Q} = \begin{matrix} x \setminus y & 1 & 2 & 3 & 4 \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 0.2 & 0.6 & 0.1 & 0.1 \\ 0.1 & 0.7 & 0.1 & 0.1 \\ 0.3 & 0.3 & 0.3 & 0.1 \end{bmatrix} \end{matrix} \begin{matrix} \xrightarrow{\times 0.2} \\ \xrightarrow{\times 0.1} \\ \xrightarrow{\times 0.7} \end{matrix} \begin{matrix} x \setminus y & 1 & 2 & 3 & 4 \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 0.04 & 0.12 & 0.02 & 0.02 \\ 0.01 & 0.07 & 0.01 & 0.01 \\ 0.21 & 0.21 & 0.21 & 0.07 \end{bmatrix} \end{matrix} = \mathbf{P}$$

$$\underline{q} = [0.26 \ 0.4 \ 0.24 \ 0.1]$$

double-check

c. Suppose the naïve decoder is used. Find the corresponding $P(\mathcal{E})$.

$$\mathbf{P} = \begin{matrix} x \setminus y & 1 & 2 & 3 & 4 \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 0.04 & 0.12 & 0.02 & 0.02 \\ 0.01 & 0.07 & 0.01 & 0.01 \\ 0.21 & 0.21 & 0.21 & 0.07 \end{bmatrix} \end{matrix}$$

$$P(\mathcal{C}) = 0.04 + 0.07 + 0.21 = 0.32$$

$$P(\mathcal{E}) = 1 - 0.32 = 0.68$$