

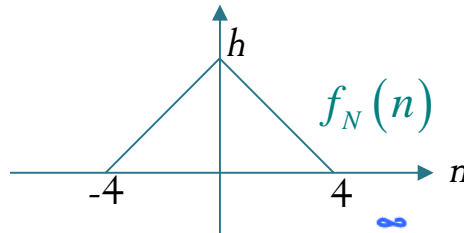
# ECS 452: In-Class Exercise # 18

## Instructions

1. Separate into groups of no more than three persons.
2. The group cannot be the same as any of your former groups for this class.
3. Only one submission is needed for each group.
4. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
5. **Do not panic.**

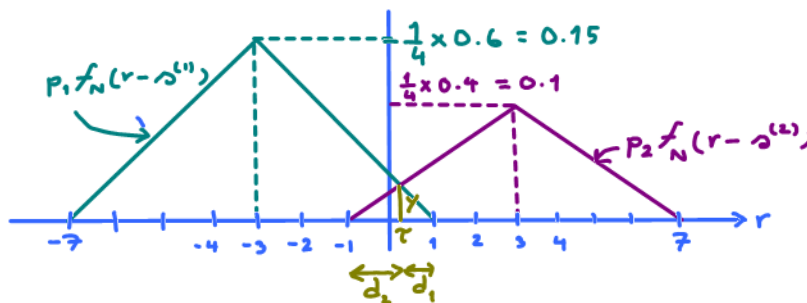
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|-------------------------|--|--|--------------------|
| Date: <b>23/05/2017</b> |  |  |                    |
| Name                    |  |  | ID (last 3 digits) |
| <b>Prapun</b>           |  |  | <b>5 5 5</b>       |
|                         |  |  |                    |
|                         |  |  |                    |

In a binary antipodal signaling scheme, the message  $S$  is randomly selected from the alphabet set  $\mathcal{S} = \{-3, 3\}$  with  $p_1 = P[S = -3] = 0.6$  and  $p_2 = P[S = 3] = 0.4$ . The message is corrupted by an independent additive noise  $N$  whose pdf is shown below:



- a. What is the value of  $h$ ? To be a pdf, we need  $\int_{-\infty}^{\infty} f_N(n) dn = 1$ .
- $$\frac{1}{2} \times 8 \times h = 1 \Rightarrow h = \frac{1}{4}$$

- b. Suppose the received symbol is  $R = r$ . Find the MAP detector  $\hat{s}_{\text{MAP}}(r)$ .



$$\begin{aligned} d_1 + d_2 &= 2 \\ \frac{2}{3}d_2 + d_2 &= 2 \\ d_2 &= 2 \times \frac{3}{5} = \frac{6}{5} = 1.2 \\ \tau &= -1 + d_2 = 0.2 \end{aligned}$$

Therefore,

$$\hat{s}_{\text{MAP}}(r) = \begin{cases} -3, & -7 < r < 0.2, \\ 3, & 0.2 < r < 7, \\ \text{any}, & \text{otherwise} \end{cases}$$

$$= \begin{cases} -3, & r < 0.2, \\ 3, & r \geq 0.2. \end{cases}$$

Use similar triangles:

$$\begin{aligned} \frac{0.15}{4} &= \frac{y}{d_1} & \frac{y}{d_2} &= \frac{0.1}{4} \\ \downarrow & & \downarrow & \\ \frac{d_1 \times 0.15}{4} &= \frac{d_2 \times 0.1}{4} \\ 3d_1 &= 2d_2 \Rightarrow d_1 = \frac{2}{3}d_2 \end{aligned}$$

- c. Evaluate the error probability of the MAP detector.

$$y = d_2 \times \frac{0.1}{4} = 1.2 \times \frac{0.1}{4} = \frac{0.12}{4} = 0.03$$

$$P(\epsilon) = \frac{1}{2} \times \underbrace{(d_1 + d_2)}_2 \times y = y = 0.03$$