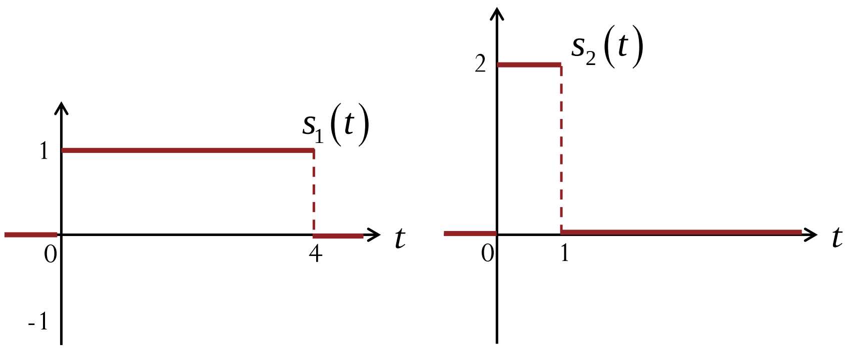
ECS 452: In-Class Exercise #16_1

Date: 11/05/2017			
Name	ID (last 3 digits)		
Prapun	5	5	5

Instructions

1. Separate into groups of no more than three persons.
2. The group cannot be the same as any of your former groups for this class.
3. Only one submission is needed for each group.
4. ***Write down all the steps*** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
5. **Do not panic.**

Consider the two signals $s_1(t)$ and $s_2(t)$ shown below.



a. Find the energy of each signal.

$$E_{s_1} = \int_{-\infty}^{\infty} s_1^2(t)\,dt = 4 \times 1 = 4$$

$$E_{s_2} = \int_{-\infty}^{\infty} s_2^2(t)\,dt = 2^2 \times 1 = 4$$

b. Find their inner product $\langle s_1(t), s_2(t) \rangle$.

$$\langle s_1, s_2 \rangle = \int_{-\infty}^{\infty} s_1(t) s_2(t)\,dt = \int_0^4 s_2(t)\,dt = \int_0^1 2\,dt = 2$$

c. Find and plot $\text{proj}_{s_1(t)}\, s_2(t)$.

$$\text{proj}_{s_1} s_2 = \frac{\langle s_2, s_1 \rangle}{\langle s_1, s_1 \rangle} s_1 = \frac{2}{4} s_1 = \frac{1}{2} s_1$$

$$= \begin{cases} 1/2, & 0 < t < 4, \\ 0, & \text{otherwise.} \end{cases}$$

Note: In the provided figure, we can't tell the values of $s_1(t)$ at $t=0$ and at $t=4$. Therefore, here, it is also ok to use "$\le$" to define the interval.

Continue on the next page

Exercise 16_2

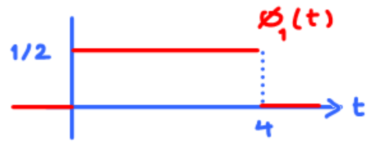
Name	ID (last 3 digits)		
Prapun	5	5	5

d. Suppose the Gram-Schmidt orthogonalization procedure (GSOP) (where the signals are applied **in the order given**) is used to find two orthonormal functions $\phi_1(t)$ and $\phi_2(t)$ that can be used as axes to represent $s_1(t)$ and $s_2(t)$.

i. Find and plot $\phi_1(t)$

$u_1(t) = s_1(t)$

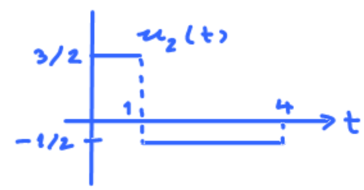
$\phi_1(t) = \dfrac{u_1(t)}{\sqrt{E_{u_1}}} = \dfrac{s_1(t)}{\sqrt{E_{s_1}}} = \dfrac{1}{2} s_1(t)$



$\Rightarrow s_1 = 2\phi_1$

ii. Find and plot $\phi_2(t)$.

$u_2(t) = s_2(t) - \text{proj}_{u_1} s_2 = s_2(t) - \frac{1}{2} s_1(t) =$



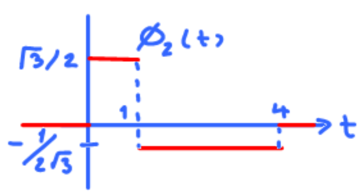
$\Rightarrow s_2 = \frac{1}{2} s_1 + u_2$

$= \frac{1}{2} 2\phi_1 + \sqrt{3}\phi_2$

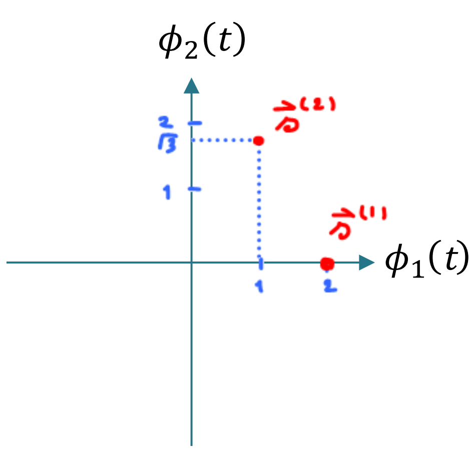
$= \phi_1 + \sqrt{3}\phi_2$

$E_{u_2} = \displaystyle\int_{-\infty}^{\infty} u_2^2(t)\,dt = \left(\frac{3}{2}\right)^2 + 3\times\left(-\frac{1}{2}\right)^2 = \frac{9}{4} + \frac{3}{4} = \frac{12}{4} = 3$

$\phi_2(t) = \dfrac{u_2(t)}{\sqrt{E_{u_2}}} = \dfrac{u_2(t)}{\sqrt{3}}$



iii. Find the two vectors, $\vec{\mathbf{s}}^{(1)}$ and $\vec{\mathbf{s}}^{(2)}$, that represent the two waveforms $s_1(t)$ and $s_2(t)$ in the new axes based on $\phi_1(t)$ and $\phi_2(t)$. Draw the corresponding constellation in the figure below.



$s_1 = 2\phi_1 \quad \Rightarrow \vec{s}^{(1)} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

$s_2 = \phi_1 + \sqrt{3}\phi_2 \quad \Rightarrow \vec{s}^{(2)} = \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}$