ECS 452: In-Class Exercise # <u>15</u>

Instructions

1. Separate into groups of no more than three persons.
2. The group cannot be the same as your former group.
3. Only one submission is needed for each group.
4. ***Write down all the steps*** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
5. **Do not panic.**

1. Suppose $\bar{\mathbf{v}} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\bar{\mathbf{u}} = \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix}$. Find

   a. $\langle \bar{\mathbf{v}}, \bar{\mathbf{u}} \rangle$

   $= \vec{u}^H \vec{v} = (-1 \quad 3 \quad -2) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = -1 + 6 - 6 = -1$

   b. $\langle \bar{\mathbf{u}}, \bar{\mathbf{u}} \rangle$

   $= \vec{u}^H \vec{u} = (-1 \quad 3 \quad -2) \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix} = (-1)^2 + 3^2 + (-2)^2 = 1 + 9 + 4 = 14$

   c. $\langle \bar{\mathbf{v}}, \bar{\mathbf{v}} \rangle$

   $= \vec{v}^H \vec{v} = (1 \quad 2 \quad 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = 1^2 + 2^2 + 3^2 = 1 + 4 + 9 = 14$

   d. $\|\bar{\mathbf{v}}\|$

   $= \sqrt{\langle \vec{v}, \vec{v} \rangle} = \sqrt{14} \approx 3.7417$

   e. $\|\bar{\mathbf{u}}\|$

   $= \sqrt{\langle \vec{u}, \vec{u} \rangle} = \sqrt{14} \approx 3.7417$

   f. $\text{proj}_{\bar{\mathbf{u}}} \bar{\mathbf{v}}$

   $= \dfrac{\langle \vec{v}, \vec{u} \rangle}{\langle \vec{u}, \vec{u} \rangle} \vec{u} = -\dfrac{1}{14} \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix} = \dfrac{1}{14} \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 1/14 \\ -3/14 \\ 1/7 \end{pmatrix}$

   Check: $\vec{e} = \vec{v} - \text{proj}_{\vec{u}} \vec{v} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} - \dfrac{1}{14} \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} = \dfrac{1}{14} \begin{pmatrix} 14 - 1 \\ 28 + 3 \\ 42 - 2 \end{pmatrix} = \dfrac{1}{14} \begin{pmatrix} 13 \\ 31 \\ 40 \end{pmatrix}$

   $\langle \vec{e}, \vec{u} \rangle = \dfrac{1}{14} (-1 \quad 3 \quad -2) \begin{pmatrix} 13 \\ 31 \\ 40 \end{pmatrix} = \dfrac{1}{14} (-13 + 93 - 80) = 0 \checkmark$

   ↳ orthogonal!