Digital Communication Systems ECS 452

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8. Optimal Detection for Additive Noise Channels
1-D Case

Office Hours:

BKD, 4th floor of Sirindhralai building

Monday	14:00-16:00
Thursday	10:30-11:30
Friday	12:00-13:00

Review: MAP decoder

3.36. A recipe for finding the MAP decoder and its corresponding error probability:

optimal

(a) Find the $\mathbf{P}$ matrix by scaling elements in each row of the $\mathbf{Q}$ matrix by their corresponding prior probability $p(x)$.

(b) Select (by circling) the maximum value in each column (for each value of y) in the $\mathbf{P}$ matrix.

- If there are multiple max values in a column, select one. This won't affect the optimality of your answer.

(i) The corresponding x value is the value of $\hat{x}$ for that y.

(ii) The sum of the selected values from the $\mathbf{P}$ matrix is $P(\mathcal{C})$.

(c) $P(\mathcal{E}) = 1 - P(\mathcal{C})$.

Review: MAP decoder

optimal

Example 3.38. Find the MAP decoder and its corresponding error probability for the DMC channel whose **Q** matrix is given by

$$\mathbf{Q} = \begin{array}{c|ccc} x \setminus y & 1 & 2 & 3 \\ \hline 0 & 0.5 & 0.2 & 0.3 \\ 1 & 0.3 & 0.4 & 0.3 \end{array}$$

$\times 0.6$, $\times 0.4$

$\hat{x}$	0	1	0
$x \setminus y$	1	2	3
0	(0.3)	0.12	(0.18)
1	0.12	(0.16)	0.12

$= \mathbf{P}$

and $\underline{\mathbf{p}} = [0.6, 0.4]$. Note that the DMC is the same as in Example 3.21 but the input probabilities are different.

$P(0)$, $P(1)$

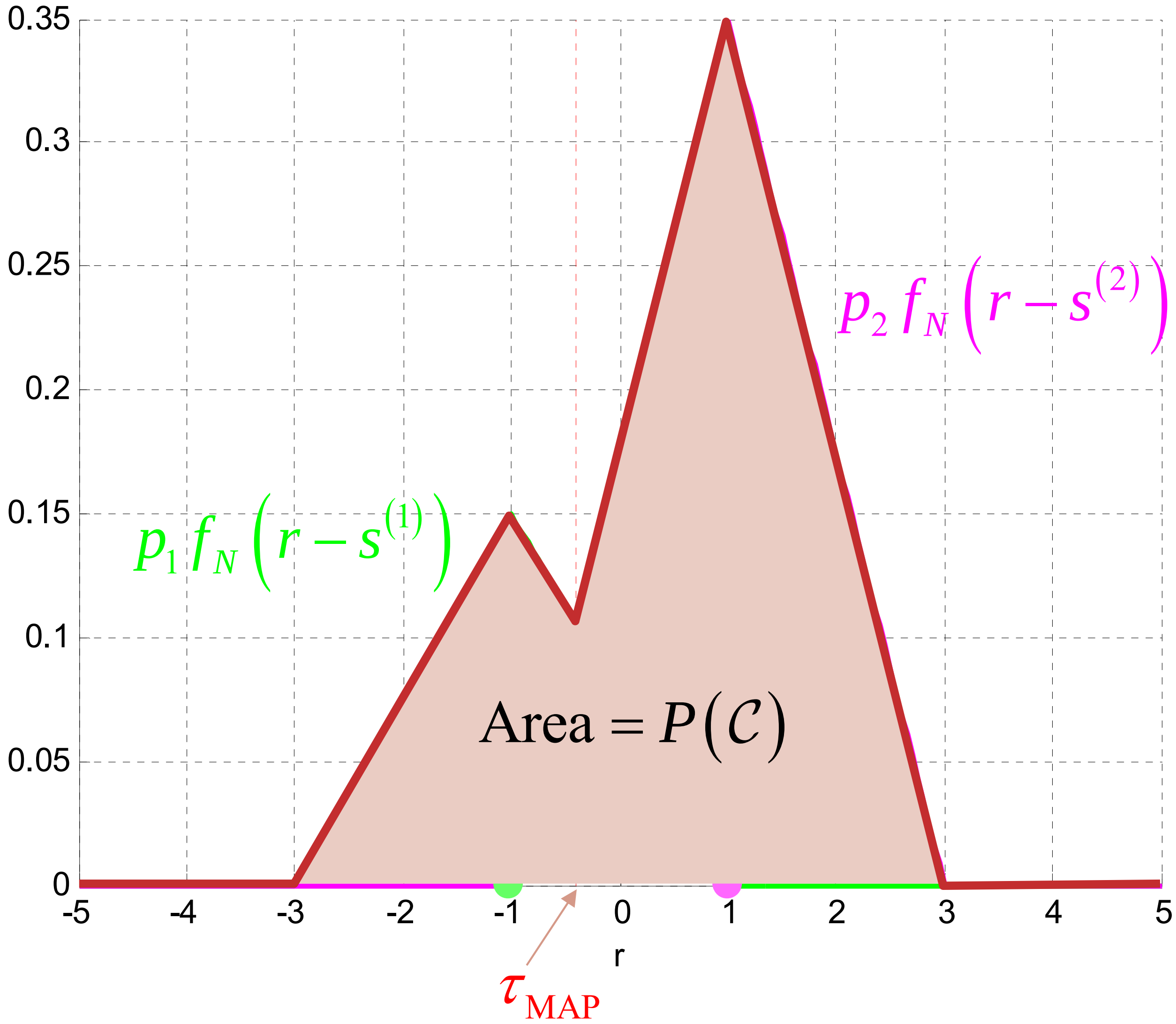
$P(C) = 0.3 + 0.16 + 0.18 = 0.64$

$P(\mathcal{E}) = 1 - 0.64 = 0.36$

y	$\hat{x}(y)$
1	0
2	1
3	0

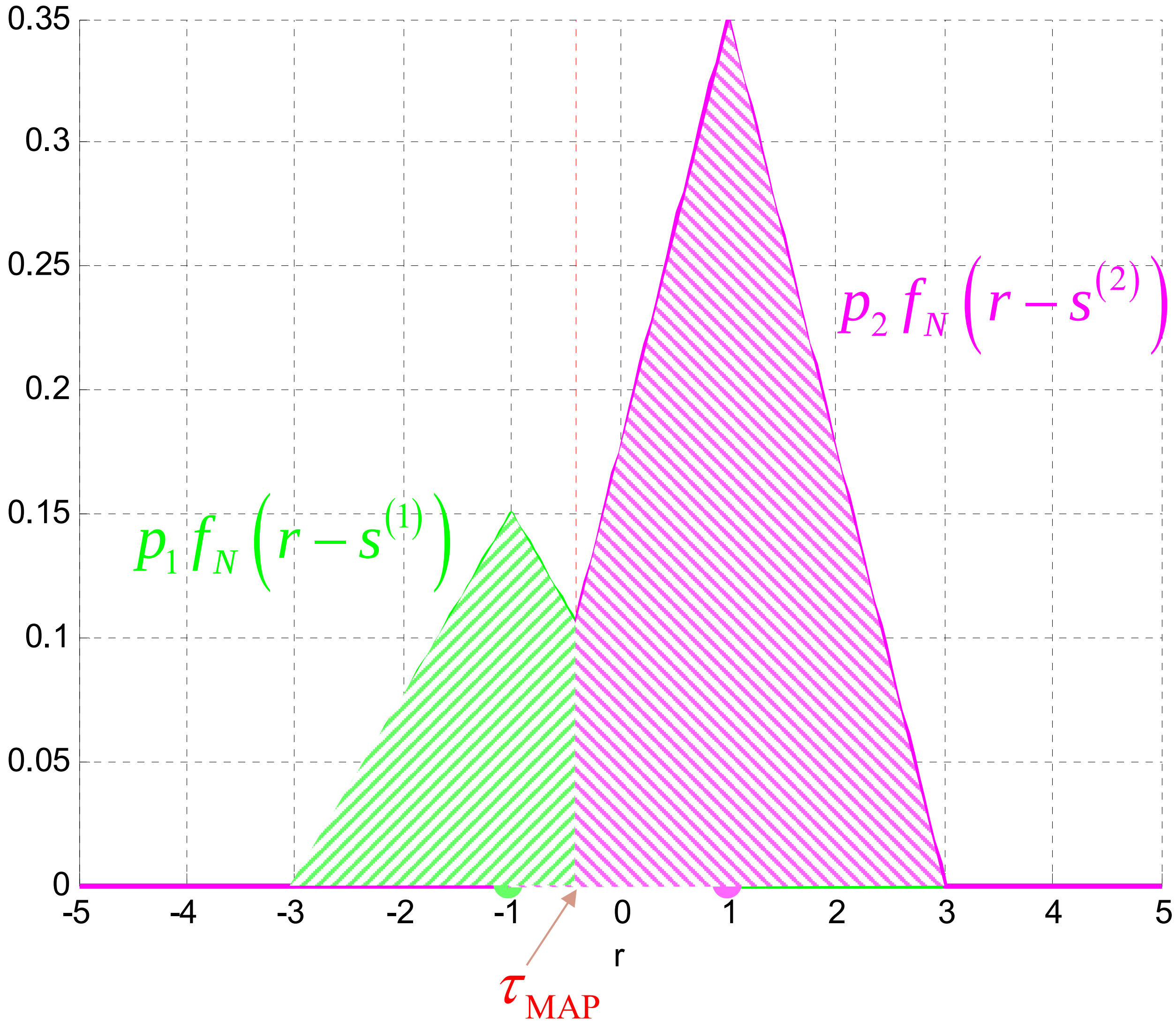
Error Probability

Ex. Binary PAM under "Triangular" Noise



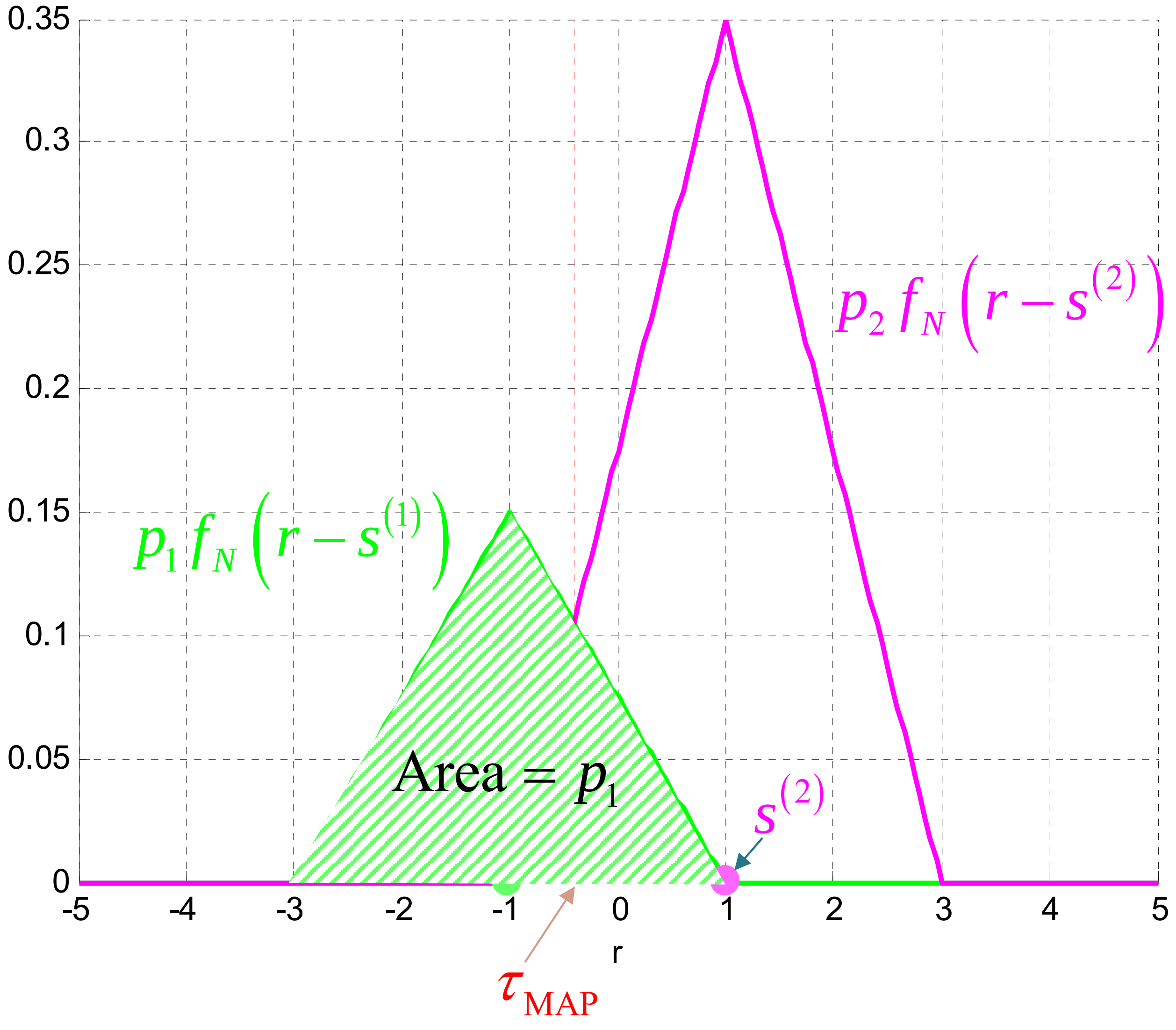
Error Probability

Ex. Binary PAM under "Triangular" Noise



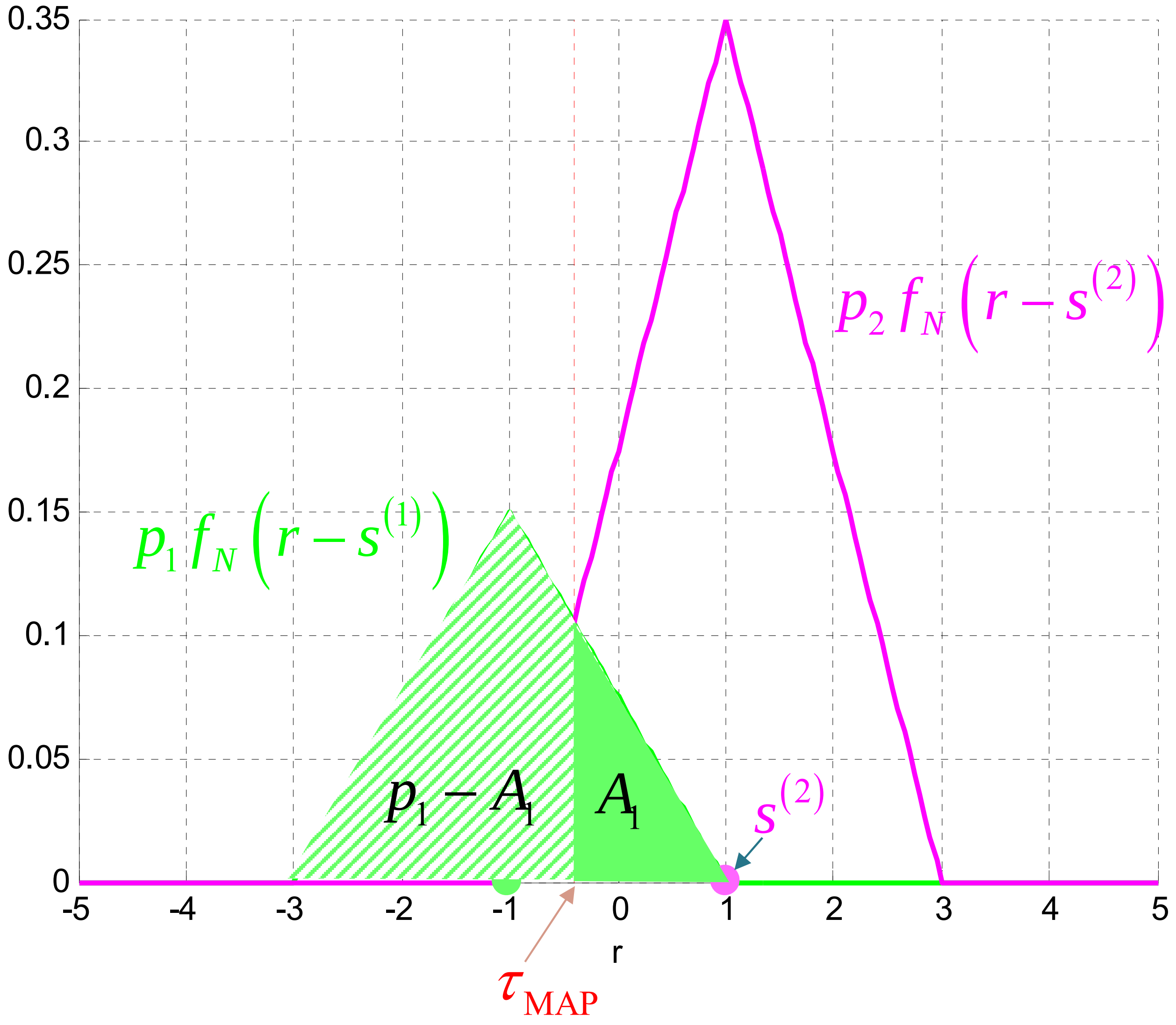
Error Probability

Ex. Binary PAM under "Triangular" Noise



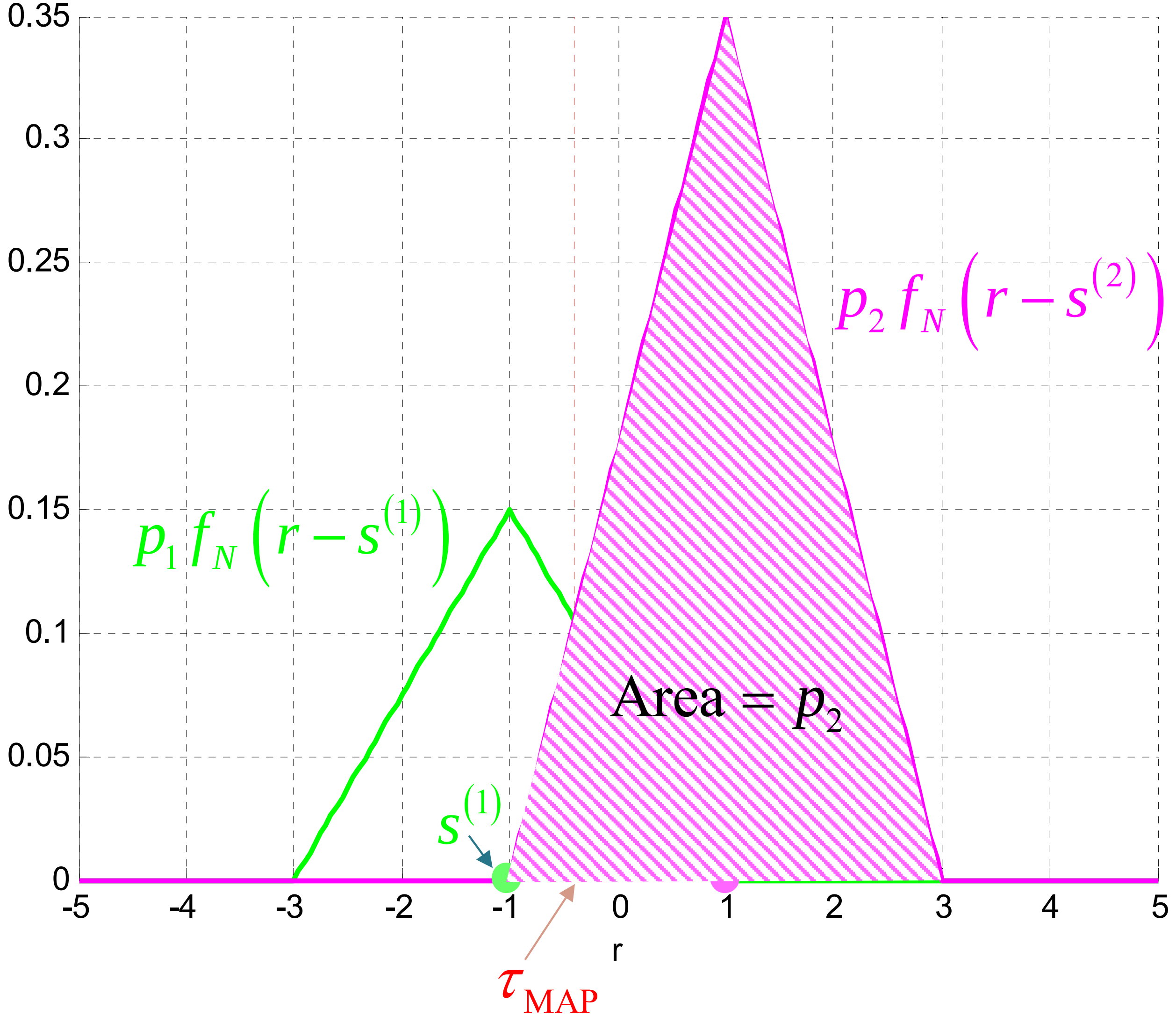
Error Probability

Ex. Binary PAM under "Triangular" Noise



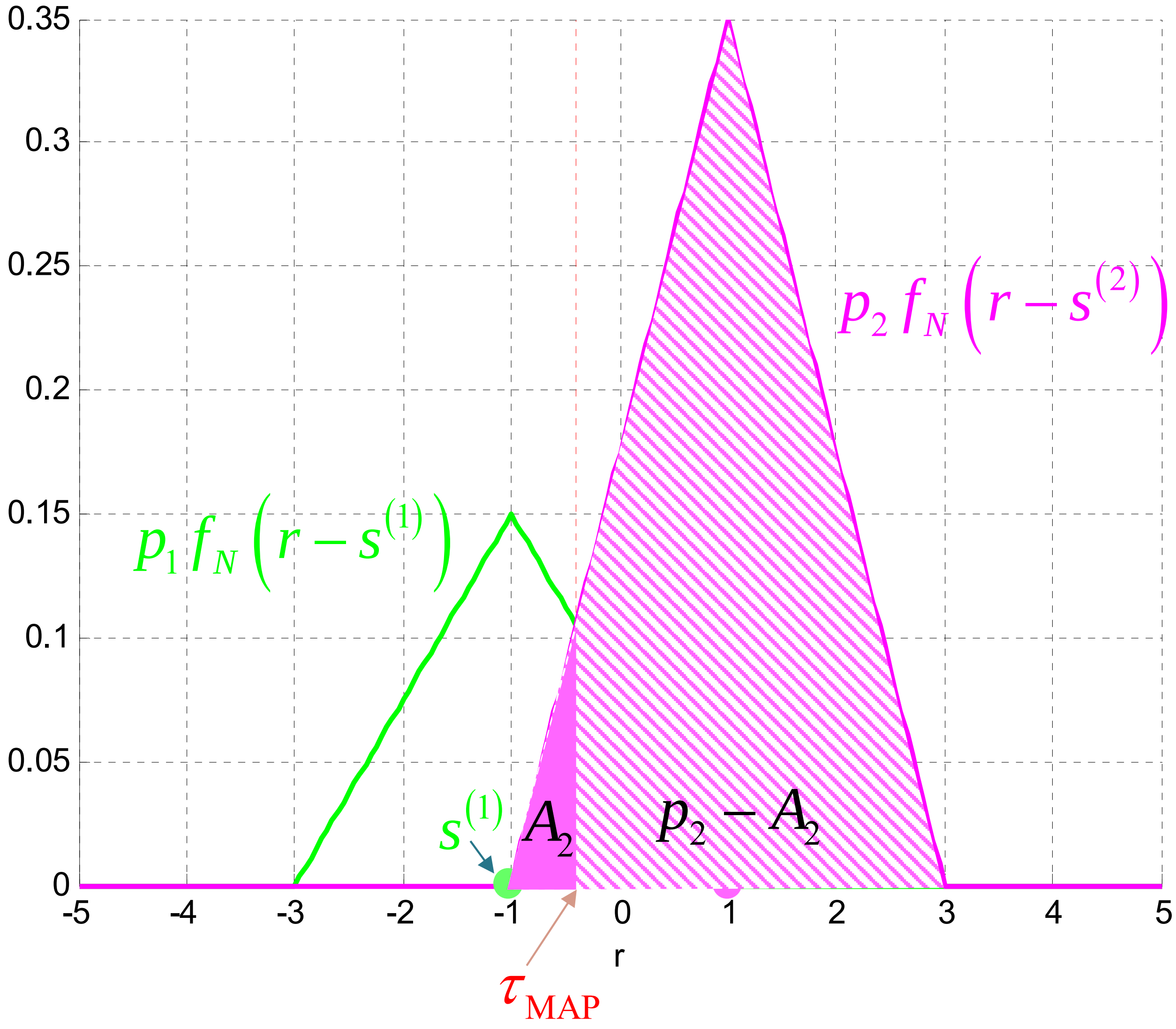
Error Probability

Ex. Binary PAM under "Triangular" Noise



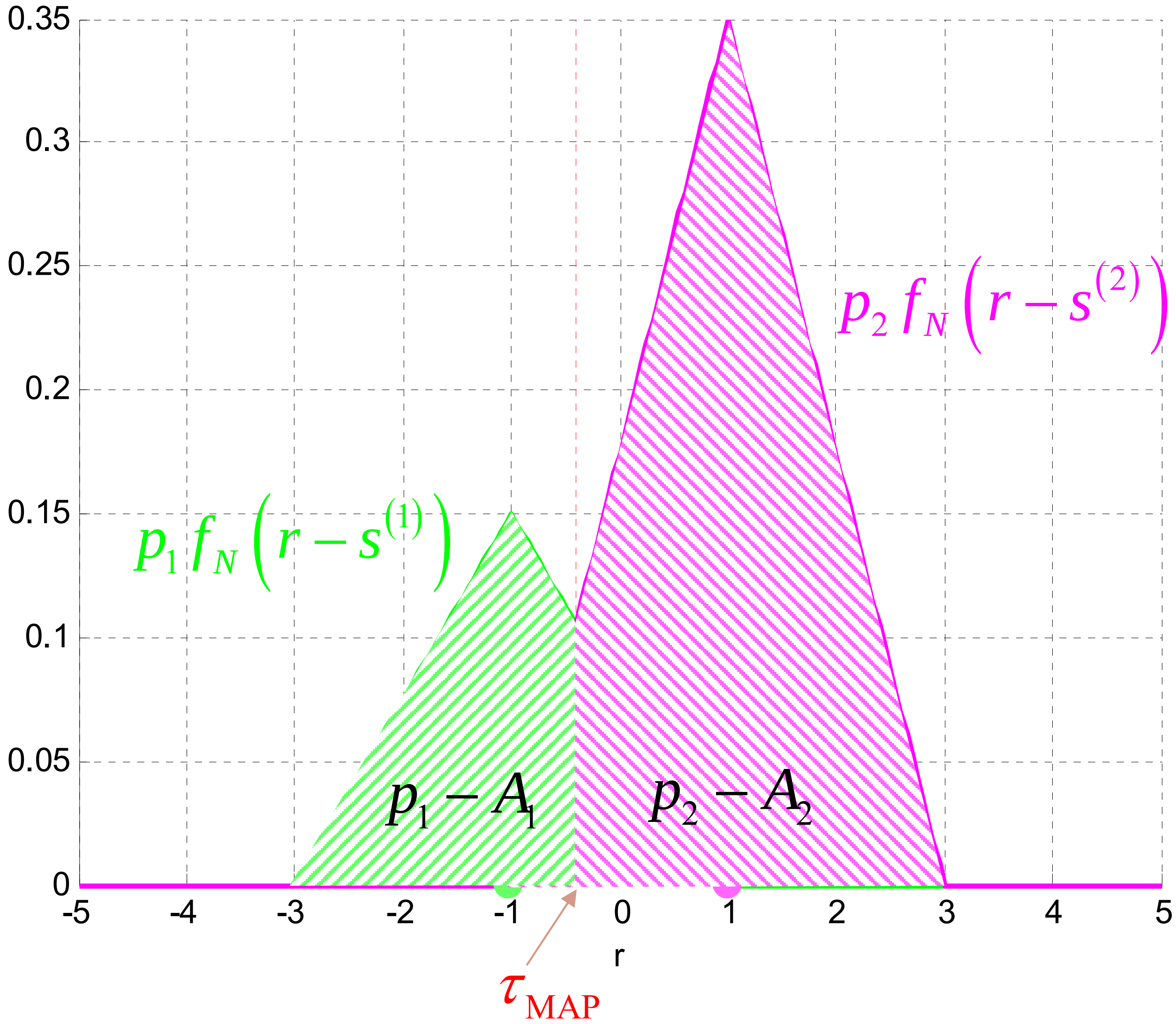
Error Probability

Ex. Binary PAM under "Triangular" Noise



Error Probability

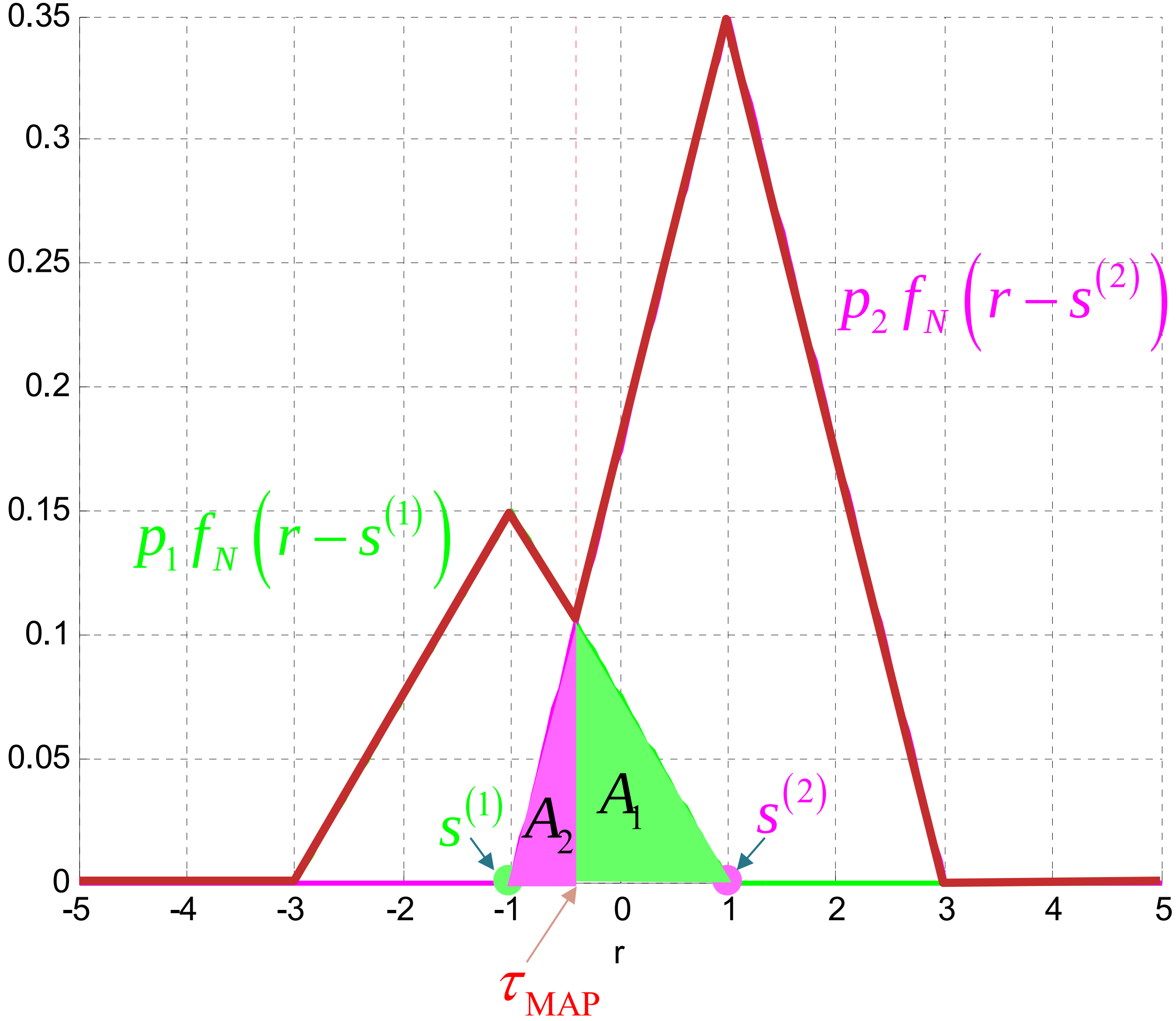
Ex. Binary PAM under "Triangular" Noise



$$P(\mathcal{C}) = (p_1 - A_1) + (p_2 - A_2)$$
$$= 1 - (A_1 + A_2)$$
$$P(\mathcal{E}) = 1 - P(\mathcal{C})$$
$$= A_1 + A_2$$

Error Probability

Ex. Binary PAM under "Triangular" Noise

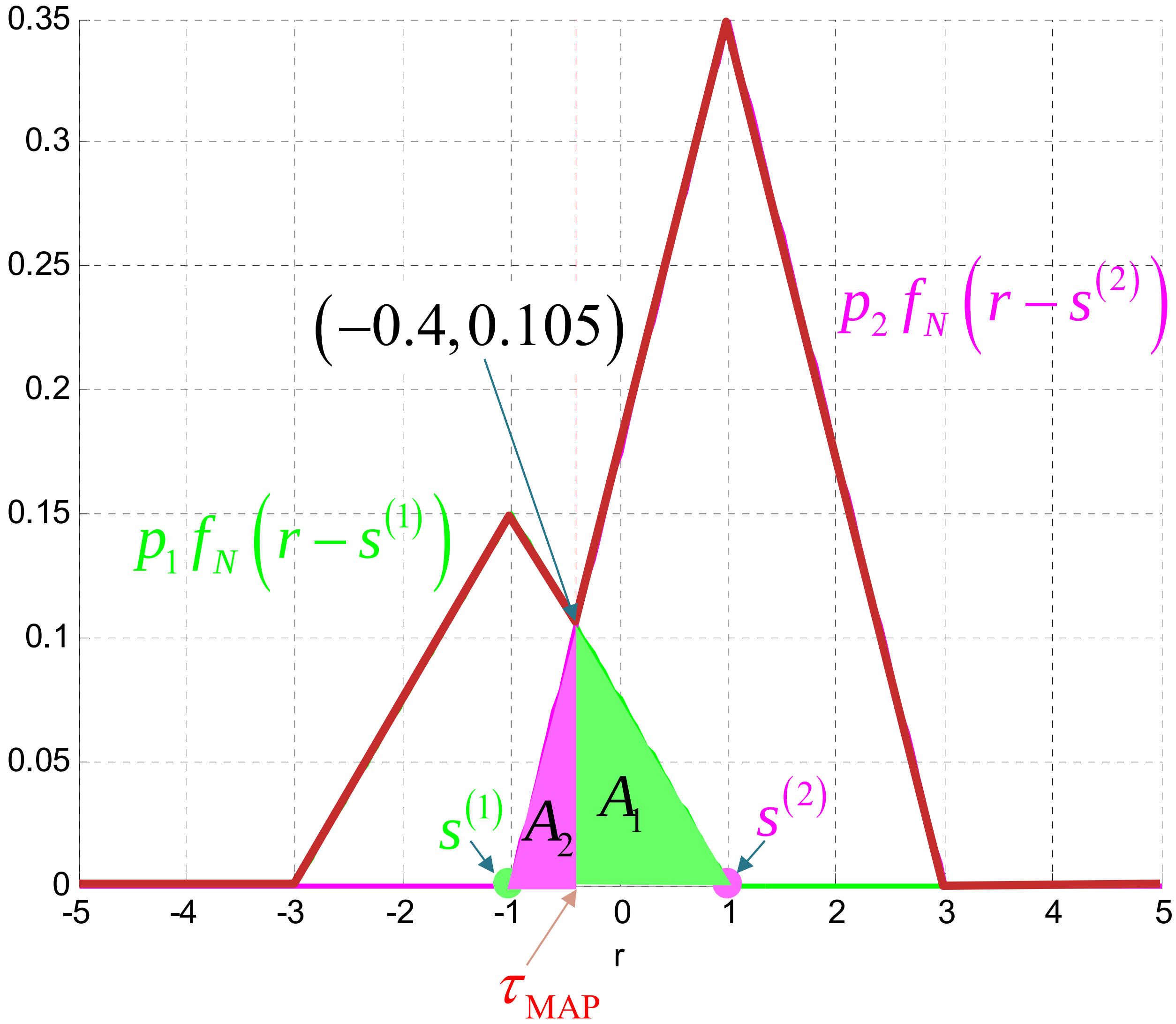


$$P(\mathcal{C}) = (p_1 - A_1) + (p_2 - A_2) = 1 - (A_1 + A_2)$$

$$P(\mathcal{E}) = 1 - P(\mathcal{C}) = A_1 + A_2$$

Error Probability

Ex. Binary PAM under "Triangular" Noise



$$P(\mathcal{C}) = (p_1 - A_1) + (p_2 - A_2) = 1 - (A_1 + A_2)$$

$$P(\mathcal{E}) = 1 - P(\mathcal{C}) = A_1 + A_2 = \frac{1}{2} \times 2 \times 0.105 = 0.105$$