ECS 452: Digital Communication Systems **2015/2**

HW 7 — Due: Not Due

Lecturer: Asst. Prof. Dr. Prapun Suksompong

Problem 1. Consider a convolutional encoder whose state diagram is given in Figure 7.1.

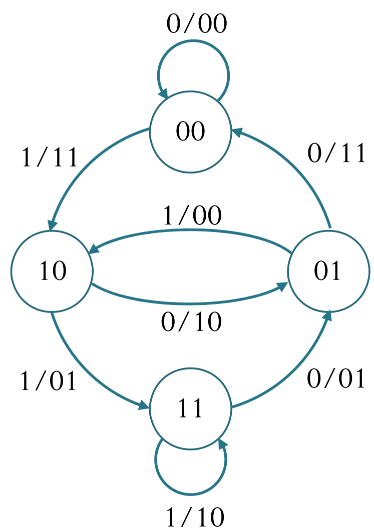

Figure 7.1: State diagram for a convolutional encoder

(a) Find the code rate

(b) Suppose the data bits (message) are 0100101. Find the corresponding codeword.

(c) Find the data vector $\underline{\mathbf{b}}$ which gives the codeword $\underline{\mathbf{x}} = [0011101111110110011]$

Problem 2. Consider a convolutional encoder whose trellis diagram is given in Figure 7.2.

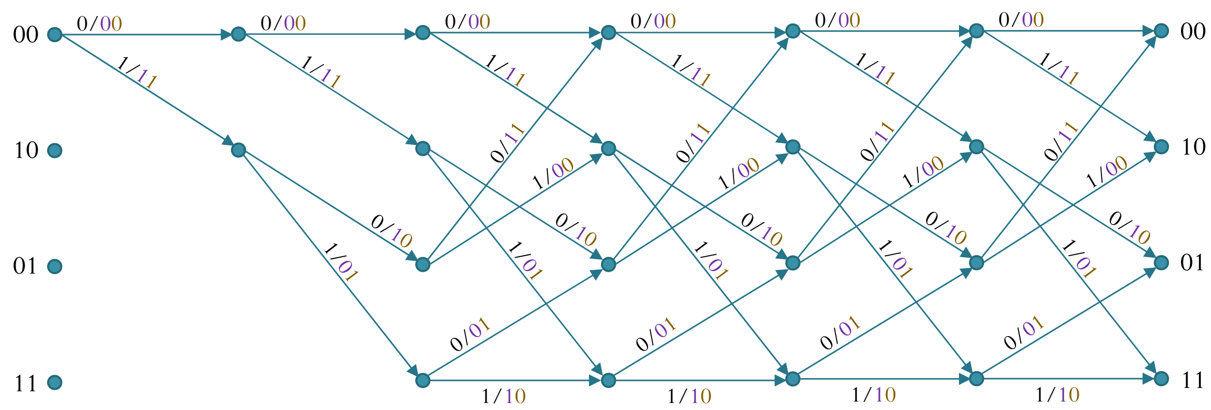

Figure 7.2: State diagram for a convolutional encoder

(a) Find the code rate

(b) Suppose the data bits (message) are $\underline{\mathbf{b}} = [0100101]$. Find the corresponding codeword $\underline{\mathbf{x}}$.

(c) Find the data vector $\underline{\mathbf{b}}$ which gives the codeword $\underline{\mathbf{x}} = [001110111110]$.

(d) Suppose that we observe $\underline{\mathbf{y}} = [001110000101]$ at the input of the minimum distance decoder. Explain why we can easily find the decoded codeword $\hat{\underline{\mathbf{x}}}$ and the decoded message $\hat{\underline{\mathbf{b}}}$ without applying the Viterbi algorithm.

(e) Suppose that we observe $\underline{\mathbf{y}} = [010101111110]$ at the input of the minimum distance decoder. Use Viterbi algorithm to find the decoded codeword $\hat{\underline{\mathbf{x}}}$ and the decoded message $\hat{\underline{\mathbf{b}}}$. Show your work on Figure 7.3 below.

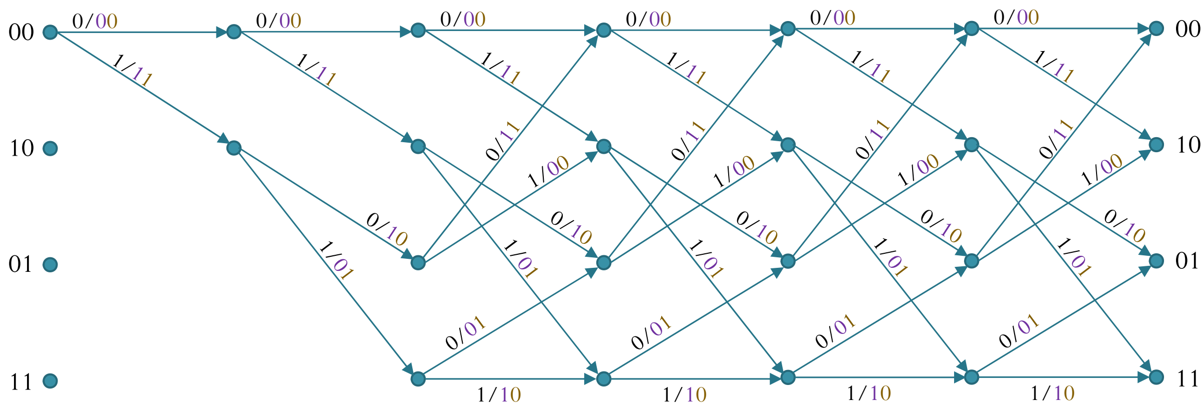

Figure 7.3: State diagram for a convolutional encoder

Make sure that all the running (cumulative) path metric are shown and the discarded branches are indicated at every steps.

Problem 3. Consider the two signals $s_1(t)$ and $s_2(t)$ shown in Figure 7.4. Note that V and T_b are some positive constants. Your answers should be given in terms of them.

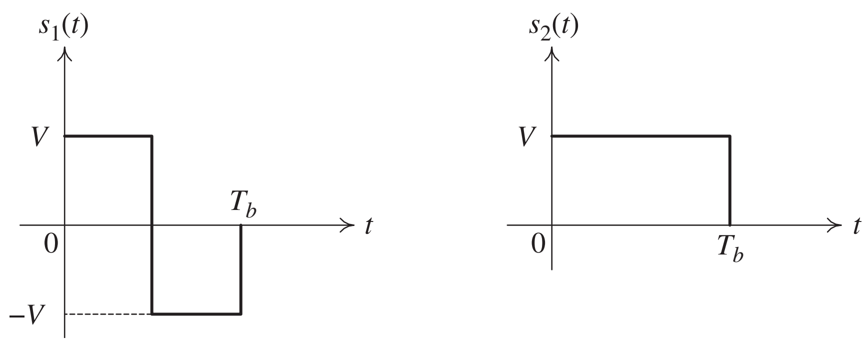

Figure 7.4: Signal set for Question 3

(a) Find the energy in each signal.

(b) Use the Gram-Schmidt orthogonalization procedure (GSOP) (where the signals are applied **<u>in the order given</u>**) to find two orthonormal functions $\phi_1(t)$ and $\phi_2(t)$ that can be used to represent $s_1(t)$ and $s_2(t)$.

(c) Find the two vectors that represent the two waveforms $s_1(t)$ and $s_2(t)$ in the new (signal) space based on the orthonormal basis found in the previous part. Draw the corresponding constellation.

Problem 4. Consider the two signals $s_1(t)$ and $s_2(t)$ shown in Figure 7.5. Note that V, α and T_b are some positive constants.

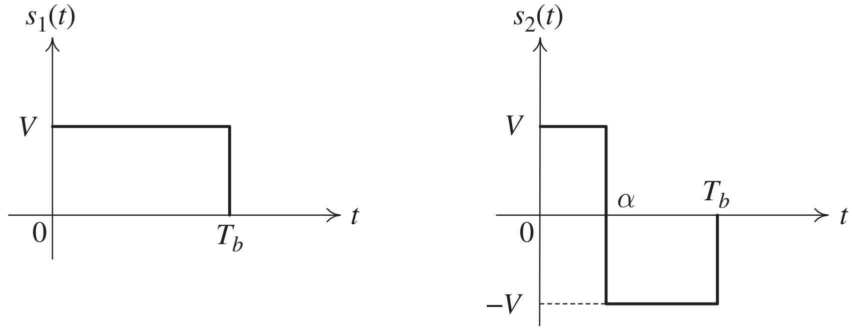

Figure 7.5: Signal set for Question 4

(a) Find the energy in each signal.

(b) Use the Gram-Schmidt orthogonalization procedure (GSOP) (where the signals are applied **in the order given**) to find two orthonormal functions $\phi_1(t)$ and $\phi_2(t)$ that can be used to represent $s_1(t)$ and $s_2(t)$.

(c) Plot $\phi_1(t)$ and $\phi_2(t)$ when $\alpha = \frac{T_b}{4}$.

(d) Find the two vectors $\boldsymbol{s}^{(1)}$ and $\boldsymbol{s}^{(2)}$ that represent the two waveforms $s_1(t)$ and $s_2(t)$ in the new (signal) space based on the orthonormal basis found in the previous part.

(e) Draw the corresponding constellation when $\alpha = \frac{T_b}{4}$.

(f) Draw $\boldsymbol{s}^{(2)}$ when $\alpha = \frac{k}{10}T_b$ where $k = 1, 2, \dots, 9$.

Problem 5. In a ternary signaling scheme, the message S is randomly selected from the alphabet set $\mathcal{S} = \{-1, 1, 4\}$ with $p_1 = P[S = -1] = 0.41$, $p_2 = P[S = 1] = 0.08$ and $p_3 = P[S = 4] = 0.51$. Find the average signal energy E_s.

Problem 6. Consider a ternary constellation. Assume that the three vectors are equiprobable.

(a) Suppose the three vectors are

$$\boldsymbol{s}^{(1)} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \boldsymbol{s}^{(2)} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, \text{ and } \boldsymbol{s}^{(3)} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

Find the corresponding average energy per symbol.

(b) Suppose we can shift the above constellation to other location; that is, suppose that the three vectors in the constellation are

$$\boldsymbol{s}^{(1)} = \begin{pmatrix} 0 - a_1 \\ 0 - a_2 \end{pmatrix}, \boldsymbol{s}^{(2)} = \begin{pmatrix} 3 - a_1 \\ 0 - a_2 \end{pmatrix}, \text{ and } \boldsymbol{s}^{(3)} = \begin{pmatrix} 3 - a_1 \\ 3 - a_2 \end{pmatrix}.$$

Find a_1 and a_2 such that corresponding average energy per symbol is minimum.

Problem 7. Prove the following facts with the help of Fourier transform.
(Hint: inner product in the frequency domain, Parseval's theorem)

(a) The energy of $p(t) = g(t)\cos(2\pi f_c t + \phi)$ is $E_g/2$.

(b) $g(t)\cos(2\pi f_c t)$ and $-g(t)\sin(2\pi f_c t)$ are orthogonal.

Is there any condition(s) on $g(t)$ for this technique to work?