**ECS 452: Digital Communication Systems** **2015/2**

# HW 4 — Due: Not Due

*Lecturer: Asst. Prof. Dr. Prapun Suksompong*

**Problem 1** (HW4-2015-2, Free)**.** In each row of the table below, compare the entropy $H(X)$ of the random variable $X$ in the first column with the entropy $H(Y)$ of the random variable $Y$ in the third column by writing ">", "=", or "<" in the second column. Watch out for approximation and round-off error.

| | | |
|---|---|---|
| $H(X)$ when $\underline{\mathbf{p}} = [0.3, 0.7]$. | | $H(Y)$ when $\underline{\mathbf{q}} = [0.8, 0.2]$. |
| $H(X)$ when $\underline{\mathbf{p}} = [0.3, 0.3, 0.4]$. | | $H(Y)$ when $\underline{\mathbf{q}} = [0.4, 0.3, 0.3]$. |
| $H(X)$ when $p(x) = \begin{cases} 0.3, & x \in \{1,2\}, \\ 0.2, & x \in \{3,4\}, \\ 0, & \text{otherwise.} \end{cases}$ | | $H(Y)$ when $\underline{\mathbf{q}} = [0.4, 0.3, 0.3]$. |

**Problem 2** (HW4-2015-2, Free)**.** Consider random variables $X$ and $Y$ whose joint pmf is given by

$$p_{X,Y}(x,y) = \begin{cases} c(x+y), & x \in \{1,3\} \text{ and } y \in \{2,4\}, \\ 0, & \text{otherwise.} \end{cases}$$

Evaluate the following quantities.

(a) $c$

(b) $H(X,Y)$

(c) $H(X)$

(d) $H(Y)$

(e) $H(X|Y)$

(f) $H(Y|X)$

(g) $I(X;Y)$

**Problem 3** (HW4-2015-2, Free)**.** Consider a pair of random variables $X$ and $Y$ whose joint pmf is given by

$$p_{X,Y}(x,y) = \begin{cases} 1/15, & x=3, y=1, \\ 2/15, & x=4, y=1, \\ 4/15, & x=3, y=3, \\ \beta, & x=4, y=3, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the value of the constant $\beta$.

(b) Are $X$ and $Y$ independent?

(c) Evaluate the following quantities.

(i) $H(X)$

(ii) $H(Y)$

(iii) $H(X,Y)$

(iv) $H(X|Y)$

(v) $H(Y|X)$

(vi) $I(X;Y)$