ECS 452: Digital Communication Systems	2015/2

HW 2 — Due: Feb 19

Lecturer: Asst. Prof. Dr. Prapun Suksompong

Instructions

(a) Solve all non-optional problems. (5 pt)

(i) Write your first name and the last three digit of your student ID on the upper-right corner of ***every*** submitted page.

(ii) For each part, write your explanation/derivation and answer in the space provided.

(b) ONE part of a question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work on all of them.

(c) Late submission will be rejected.

(d) **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Problem 1. In this question, each output string from a DMS is encoded by the following source code:

x	Codeword $c(x)$
'a'	1
'd'	01
'e'	0000
'i'	001
'o'	00010
'u'	00011

(a) Is the code prefix-free?

(b) Is the code uniquely decodable?

(c) Suppose the DMS produces the string 'audio'. Find the output of the source encoder.

(d) Suppose the output of the source encoder is
0 0 0 1 0 0 1 0 0 0 0 0 0 1 0 1 0 0 0 0 1 0 1 0 0 1 0 0 0 0

Find the corresponding source string produced by the DMS. Use "/" to indicate the locations where the sting above is split into codewords.

Problem 2. A DMC has $\mathcal{X} = \{0, 1\}$ and $\mathcal{Y} = \{1, 2, 3\}$. The following decoding table is used to decode the channel output.

y	$\hat{x}(y)$
1	0
2	1
3	0

Suppose the channel output string is 2122132211331221221 32.

(a) Find the corresponding decoded string.

(b) Suppose the channel input string is produced from an ASCII source encoder by the command `dec2bin(SourceString,7)` in MATLAB. Assume that there is no channel decoding error. Find the corresponding source string.

Problem 3 (HW2-2015-2)**.** Consider a BSC whose crossover probability for each bit is $p = 0.35$. Suppose $P[X = 0] = 0.45$.

(a) Draw the channel diagram.

(b) Find the channel matrix $\mathbf{Q}$.

(c) Find the joint pmf matrix $\mathbf{P}$.

(d) Find the row vector $\underline{\mathbf{q}}$ which contains the pmf of the channel output Y.

(e) We now analyze the performance of all four reasonable detectors for this binary channel. Complete the table below:

$\hat{x}(y)$	$P\left[\hat{X}=0\,\middle\|\,X=0\right]$	$P\left[\hat{X}=1\,\middle\|\,X=1\right]$	$P(\mathcal{C})$	$P(\mathcal{E})$
y				
$1-y$				
1				
0				

(f) Find the MAP detector and its error probability.

(g) Find the ML detector and its error probability.

Problem 4 (HW2-2015-2)**.** Consider a DMC whose $\mathcal{X} = \{1, 2, 3\}$, $\mathcal{Y} = \{1, 2, 3\}$, and $\mathbf{Q} = \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.2 & 0.6 \end{bmatrix}$. Suppose the input probability vector is $\underline{\mathbf{p}} = [0.2, 0.4, 0.4]$.

(a) Find the joint pmf matrix $\mathbf{P}$.

(b) Find the row vector $\underline{\mathbf{q}}$ which contains the pmf of the channel output Y.

(c) Find the error probability of the naive decoder.

(d) Find the error probability of the (DIY) decoder $\hat{x}(y) = 4 - y$.

(e) Find the MAP detector and its error probability.

(f) Find the ML detector and its error probability.

Problem 5 (HW2-2015-2)**.** Consider a BAC whose $Q(1|0) = 0.35$ and $Q(0|1) = 0.55$. Suppose $P[X = 0] = 0.4$.

(a) Draw the channel diagram.

(b) Find the joint pmf matrix $\mathbf{P}$.

(c) Find the row vector $\underline{\mathbf{q}}$ which contains the pmf of the channel output Y.

(d) We now analyze the performance of all four reasonable detectors for this binary channel. Complete the table below:

$\hat{x}(y)$	$P\left[\hat{X} = 0 \,\middle|\, X = 0\right]$	$P\left[\hat{X} = 1 \,\middle|\, X = 1\right]$	$P(\mathcal{C})$	$P(\mathcal{E})$
y				
$1-y$				
1				
0				

(e) Find the MAP detector and its error probability.

(f) Find the ML detector and its error probability.

Problem 6 (HW2-2015-2)**.** (Optional) Consider a DMC whose samples of input X and output Y are recorded as row vectors $\underline{\mathbf{x}}$ and $\underline{\mathbf{y}}$ in the file `HW_DMC_Channel_Data.mat`. Write MATLAB script which uses the recorded information to estimate the quantities below. Note that most of these can be solved by appropriate parts of the m-files posted on the course web site.

(a) The support $\mathcal{X}$ of X.

(b) The support $\mathcal{Y}$ of Y.

(c) The row vector $\underline{\mathbf{p}}$ which contains the pmf of X.

(d) The $\mathbf{Q}$ matrix.

(e) The row vector $\underline{\mathbf{q}}$ which contains the pmf of Y. Do this using two methods:

  (i) Count directly from the observed values of Y.

  (ii) Use the estimated values of $\underline{\mathbf{p}}$ and $\mathbf{Q}$.

(f) The error probability when the naive decoder is used. Do this using two methods:

  (i) Directly construct $\underline{\hat{\mathbf{x}}}$ from $\underline{\mathbf{y}}$. Then, compare $\underline{\hat{\mathbf{x}}}$ and $\underline{\mathbf{x}}$.

  (ii) Use the estimated values of $\underline{\mathbf{p}}$ and $\mathbf{Q}$.

(g) The error probability when the MAP decoder is used. Do this using two methods:

  (i) First find the MAP decoder table using the estimated values of $\underline{\mathbf{p}}$ and $\mathbf{Q}$. Then, construct $\underline{\hat{\mathbf{x}}}$ from $\underline{\mathbf{y}}$ according to the decoder table. Finally, compare $\underline{\hat{\mathbf{x}}}$ and $\underline{\mathbf{x}}$.

  (ii) Use the estimated values of $\underline{\mathbf{p}}$ and $\mathbf{Q}$ to directly calculate the error probability.

(h) The error probability when the ML decoder is used. Do this using two methods:

  (i) First find the ML decoder table using the estimated value of $\mathbf{Q}$. Then, construct $\underline{\hat{\mathbf{x}}}$ from $\underline{\mathbf{y}}$ according to the decoder table. Finally, compare $\underline{\hat{\mathbf{x}}}$ and $\underline{\mathbf{x}}$.

  (ii) Use the estimated values of $\underline{\mathbf{p}}$ and $\mathbf{Q}$ to directly calculate the error probability.