ECS 452: Digital Communication Systems **2014/1**

HW 7 — Due: Not Due

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Problem 1. Suppose $s_1(t) = \text{sinc}(5t)$ and $s_2(t) = \text{sinc}(7t)$. Note that in this class, we define $\text{sinc}(x) = \frac{\sin x}{x}$. Find

(a) E_{s_1},

(b) E_{s_2}, and

(c) $\langle s_1(t), s_2(t) \rangle$.

Hint: Evaluate the above quantities in the frequency domain.

Problem 2. In a binary antipodal signaling scheme, the message S is randomly selected from the alphabet set $\mathcal{S} = \{-3, 3\}$ with $p_1 = P[S = -3] = 0.3$ and $p_2 = P[S = 3] = 0.7$. The message is corrupted by an independent additive noise N which is uniform on $[-4, 4]$.

(a) Draw the decision regions of the MAP detector.

(b) Find the corresponding error probability for each of the following detectors

(i) $\hat{s}(r) = \begin{cases} -3, & r < -3, \\ 3, & r \geq -3. \end{cases}$

(ii) $\hat{s}(r) = \begin{cases} -3, & r \geq -1, \\ 3, & r < -1. \end{cases}$

(iii) $\hat{s}(r) = \begin{cases} -3, & |r| \geq 2, \\ 3, & |r| < 2. \end{cases}$

(c) Find the ML detector and calculate its error probability.

Problem 3. In a ternary signaling scheme, the message S is randomly selected from the alphabet set $\mathcal{S} = \{-1, 1, 4\}$ with $p_1 = P[S = -1] = 0.3 = p_2 = P[S = 1]$ and $p_3 = P[S = 4] = 0.4$. The message is corrupted by an independent additive Gaussian noise $N \sim \mathcal{N}(0, 2)$.

(a) Find the average signal energy[1] E_s.

[1]Same as "average symbol energy" or "average energy per symbol" or "average energy per signal"

(b) Find the MAP detector $\hat{s}_{\text{MAP}}(r)$.

(c) Indicate the decision regions of the MAP detector in part (b).

(d) Evaluate the error probability of the MAP detector.

Problem 4. In a **standard** quaternary signaling scheme, the message S is equiprobably selected from the alphabet set $\mathcal{S} = \left\{-\frac{3d}{2}, -\frac{d}{2}, \frac{d}{2}, \frac{3d}{2}\right\}$. The message is corrupted by an independent additive exponential noise N whose pdf is

$$f_N(n) = \begin{cases} \lambda e^{-\lambda n}, & n \ge 0, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the average symbol energy.

(b) Find the average energy per bit.

(c) Find the MAP detector $\hat{s}_{\text{MAP}}(r)$.

(d) Evaluate the error probability of the MAP detector.

(e) Let $\lambda = \frac{1}{\sigma}$. (This is to set $\operatorname{Var} N = \sigma^2$ as in the case for Gaussian noise.) Plot $\frac{E_b}{\sigma^2}$ vs. probability of error $P(\mathcal{E})$. Consider $\frac{E_b}{\sigma^2}$ from -30 to 10 dB.

Problem 5. Construct a generator matrix $\mathbf{G}$ and a corresponding parity check matrix $\mathbf{H}$ for a (15,11) Hamming code.