ECS 452: Digital Communication Systems **2014/1**

HW 5 — Due: Nov 27

Lecturer: Asst. Prof. Dr. Prapun Suksompong

Instructions

(a) ONE part of a question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work on all of them. The last problem is optional.

(b) It is important that you try to solve all problems. (5 pt)

(c) Late submission will be heavily penalized.

(d) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Problem 1. Consider four vectors

$$\boldsymbol{v}^{(1)} = \begin{pmatrix} 1+j \\ 1-j \\ 0 \end{pmatrix}, \boldsymbol{v}^{(2)} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \boldsymbol{v}^{(3)} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \text{ and } \boldsymbol{v}^{(4)} = \begin{pmatrix} -1 \\ -1 \\ -j \end{pmatrix}.$$

(a) Use the Gram-Schmidt orthogonalization procedure (GSOP) (where the vectors are applied **in the order given**) to the orthonormal vectors $\boldsymbol{e}^{(1)}$, $\boldsymbol{e}^{(2)}$, ... that can be used to represent $\boldsymbol{v}^{(1)}$, $\boldsymbol{v}^{(2)}$, $\boldsymbol{v}^{(3)}$, and $\boldsymbol{v}^{(4)}$.

(b) Use the orthonormal vectors from the previous part to construct the matrix $E = \left[\boldsymbol{e}^{(1)}\ \boldsymbol{e}^{(2)}\ \cdots\right]$. Find the matric C such that $V = EC$ where $V = \left[\boldsymbol{v}^{(1)}\ \boldsymbol{v}^{(2)}\ \boldsymbol{v}^{(3)}\ \boldsymbol{v}^{(4)}\right]$.

Problem 2. Consider the two signals $s_1(t)$ and $s_2(t)$ shown in Figure 5.1. Note that V and T_b are some positive constants. Your answers should be given in terms of them.

(a) Find the energy in each signal.

(b) Use the Gram-Schmidt orthogonalization procedure (GSOP) (where the signals are applied **in the order given**) to find two orthonormal functions $\phi_1(t)$ and $\phi_2(t)$ that can be used to represent $s_1(t)$ and $s_2(t)$.

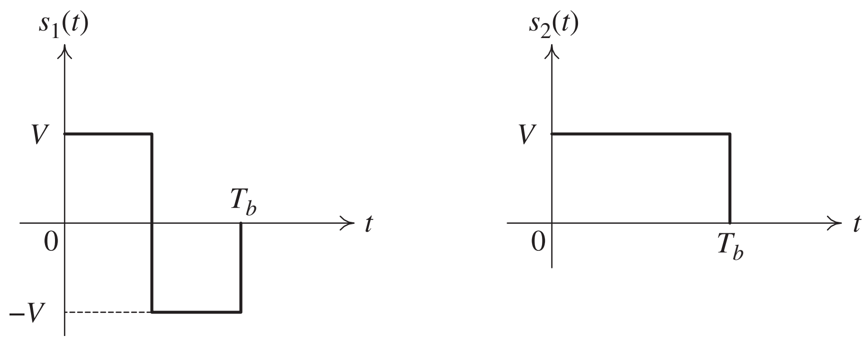

Figure 5.1: Signal set for Question 2

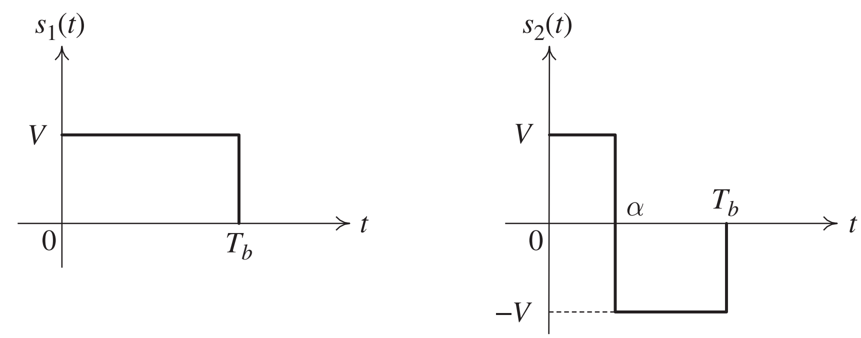

Figure 5.2: Signal set for Question 3

(c) Find the two vectors that represent the two waveforms $s_1(t)$ and $s_2(t)$ in the new (signal) space based on the orthonormal basis found in the previous part. Draw the corresponding constellation.

Problem 3. Consider the two signals $s_1(t)$ and $s_2(t)$ shown in Figure 5.2. Note that V, α and T_b are some positive constants.

(a) Find the energy in each signal.

(b) Use the Gram-Schmidt orthogonalization procedure (GSOP) (where the signals are applied **in the order given**) to find two orthonormal functions $\phi_1(t)$ and $\phi_2(t)$ that can be used to represent $s_1(t)$ and $s_2(t)$.

(c) Plot $\phi_1(t)$ and $\phi_2(t)$ when $\alpha = \frac{T_b}{4}$.

(d) Find the two vectors $\boldsymbol{s}^{(1)}$ and $\boldsymbol{s}^{(2)}$ that represent the two waveforms $s_1(t)$ and $s_2(t)$ in the new (signal) space based on the orthonormal basis found in the previous part.

(e) Draw the corresponding constellation when $\alpha = \frac{T_b}{4}$.

(f) Draw $\boldsymbol{s}^{(2)}$ when $\alpha = \frac{k}{10}T_b$ where $k = 1, 2, \ldots, 9$.