ECS 452: Digital Communication Systems	2014/1

HW 4 — Due: Nov 7

Lecturer: Asst. Prof. Dr. Prapun Suksompong

Instructions

(a) ONE part of a question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work on all of them. The last problem is optional.

(b) It is important that you try to solve all problems. (5 pt)

(c) Late submission will be heavily penalized.

(d) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

(e) You should be able to work on the first four problems without the help of MATLAB. (In the exam room, you will have only a calculator; so it is best to figure out how to work without MATLAB.)

(f) For problem that uses MATLAB, you must also print and submit your m-file along with the numerical results from the command window. Don't forget to annotate your results so that it is clear where the answer for each part is. Also, in the beginning part of your code, there should be one commented line that indicates your name and student ID.

Problem 1. For each of the DMCs whose corresponding transition probability matrices $\mathbf{Q}$ are specified below, (i) draw the channel diagram and (ii) compute its capacity C and the corresponding $\underline{\mathbf{p}}$ that achieves it.

(a)
$$\mathbf{Q} = \begin{bmatrix} 1/3 & 2/3 \\ 2/3 & 1/3 \end{bmatrix}$$

(b)
$$\mathbf{Q} = \begin{bmatrix} 0 & 1/5 & 4/5 & 0 & 0 \\ 2/3 & 0 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 0 & 1. \end{bmatrix}$$

(c)
$$\mathbf{Q} = \begin{bmatrix} 1/3 & 2/3 \\ 1/3 & 2/3 \end{bmatrix}$$

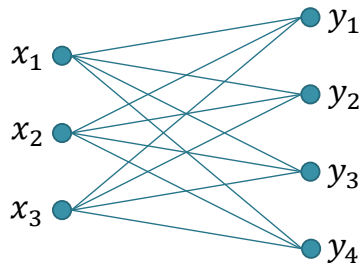**Problem 2.** The channel diagram for a DMC is shown in Figure 4.1.

Figure 4.1: Channel diagram for Problem 2.

(a) What are the dimensions of its matrix $\mathbf{Q}$?

(b) Is it possible to find appropriate values for the $\mathbf{Q}$ matrix of this DMC such that $C = 1.6$ bpcu? If so, give an example of such $\mathbf{Q}$.

(c) Is it possible to find appropriate values for the $\mathbf{Q}$ matrix of this DMC such that $C = \log_2 3$ bpcu? If so, give an example of such $\mathbf{Q}$.

Problem 3. On many occasions, we have to work with a DMC whose $\mathbf{Q}$ matrix ***does not match*** any of our special cases discussed in class. As discussed in class, capacity calculation for these more general cases can be done via treating them as optimization problems in calculus. When $|\mathcal{X}| = 2$, $\mathbf{p} = [p_0 \ 1 - p_0]$. So, the optimization is over one variable. In general, when $|\mathcal{X}| = n$, the optimization is over $n - 1$ variables.

For each of the DMCs whose transition probability matrices $\mathbf{Q}$ are specified below, (i) draw the channel diagram and (ii) compute its capacity C and the corresponding $\underline{\mathbf{p}}$ that achieves it by first calculating $I(X;Y)$ for an arbitrary $\underline{\mathbf{p}}$ (with appropriate dimension) and then set appropriate (partial) derivative(s) to 0 to solve for $\underline{\mathbf{p}}$.

(a)
$$\mathbf{Q} = \left[\begin{array}{ccc} 1/3 & 2/3 & 0 \\ 0 & 2/3 & 1/3 \end{array} \right]$$

(b)
$$\mathbf{Q} = \left[\begin{array}{cc} 1 & 0 \\ 1/2 & 1/2 \\ 1/2 & 1/2 \end{array} \right]$$

Hint: Let $\underline{\mathbf{p}} = [p_1, p_2, p_3]$. Note that the values of p_2 and p_3 only affect $I(X,Y)$ through $p_2 + p_3$, which is $1 - p_1$. So, we are back to optimizing over only one variable.

Problem 4. The selected examples of the $\mathbf{Q}$ matrices presented in class are not the only examples in which capacity values can be found without solving the full-blown optimization problems (from which you have suffered in Problem 3). Here is one more example. Consider a DMC whose transition probability matrix $\mathbf{Q}$ is

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1/3 & 2/3 \end{bmatrix}.$$

(a) Draw the channel diagram.

(b) Compute its capacity C and the corresponding $\underline{\mathbf{p}}$ that achieves it.

Hint: Bound the capacity from the dimensions of $\mathbf{Q}$ then try to find a $\underline{\mathbf{p}}$ that achieves the bound. Note that without the last row in the matrix $\mathbf{Q}$, the channel is noiseless. One may also eliminate a "useless" channel input by not using it at all.

Problem 5 (Blahut-Arimoto algorithm)**.** A `MATLAB` function `capacity_blahut` is provided on the course web site. It calculates the capacity $C = \max_{\underline{\mathbf{p}}} I(\underline{\mathbf{p}}, \mathbf{Q})$ and the corresponding capacity-achieving input pmf $\underline{\mathbf{p}}$ using Blahut-Arimoto algorithm.

Write a MATLAB script to check your answers in all earlier problems with the help of the provided `MATLAB` function.