

HW ~~Solution~~ 2 — Due: Sep 26

Lecturer: Asst. Prof. Dr. Prapun Suksompong

**Instructions**

- (a) ONE part of a question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work on all of them. The last problem is optional.
- (b) It is important that you try to solve all problems. (5 pt)
- (c) Late submission will be heavily penalized.
- (d) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
- (e) For problem that uses **MATLAB**, you must also print and submit your m-file along with the numerical results from the command window. Don't forget to annotate your results so that it is clear where the answer for each part is. Also, in the beginning part of your code, there should be one commented line that indicates your name and student ID.

**Problem 1.** Consider a BSC whose crossover probability for each bit is  $p = 0.35$ . Suppose  $P[X = 0] = 0.45$ .

- (a) Draw the channel diagram.
- (b) Find the joint pmf matrix  $\mathbf{P}$ .
- (c) Find the row vector  $\mathbf{q}$  which contains the pmf of the channel output  $Y$ .
- (d) We now analyze the performance of all four reasonable detectors for this binary channel. Complete the table below:

$\hat{x}(y)$	$P[\hat{X} = 0   X = 0]$	$P[\hat{X} = 1   X = 1]$	$P(\mathcal{C})$	$P(\mathcal{E})$
$y$				
$1 - y$				
1				
0				

- (e) Find the MAP detector and its error probability.

- (f) Find the ML detector and its error probability.

**Problem 2.** Consider a DMC whose  $\mathcal{X} = \{1, 2, 3\}$ ,  $\mathcal{Y} = \{1, 2, 3\}$ , and  $\mathbf{Q} = \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.2 & 0.6 \end{bmatrix}$ .

Suppose the prior probability vector is  $\underline{p} = [0.2, 0.4, 0.4]$ .

- Find the joint pmf matrix  $\mathbf{P}$ .
- Find the row vector  $\underline{\mathbf{q}}$  which contains the pmf of the channel output  $Y$ .
- Find the error probability of the naive decoder.
- Find the error probability of the (DIY) decoder  $\hat{x}(y) = 4 - y$ .
- Find the MAP detector and its error probability.
- Find the ML detector and its error probability.

**Problem 3.** Consider a DMC whose samples of input  $X$  and output  $Y$  are recorded as row vectors  $\underline{\mathbf{x}}$  and  $\underline{\mathbf{y}}$  in the file `HW_DMC_Channel_Data.mat`. Write MATLAB script which uses the recorded information to estimate the following quantities:

- The support  $\mathcal{X}$  of  $X$ .
- The support  $\mathcal{Y}$  of  $Y$ .
- The row vector  $\underline{\mathbf{p}}$  which contains the pmf of  $X$ .
- The  $\mathbf{Q}$  matrix.
- The row vector  $\underline{\mathbf{q}}$  which contains the pmf of  $Y$ . Do this using two methods:
  - Count directly from the observed values of  $Y$ .
  - Use the estimated values of  $\underline{\mathbf{p}}$  and  $\mathbf{Q}$ .
- The error probability when the naive decoder is used. Do this using two methods:
  - Directly construct  $\underline{\hat{\mathbf{x}}}$  from  $\underline{\mathbf{y}}$ . Then, compare  $\underline{\hat{\mathbf{x}}}$  and  $\underline{\mathbf{x}}$ .
  - Use the estimated values of  $\underline{\mathbf{p}}$  and  $\mathbf{Q}$ .
- The error probability when the MAP decoder is used. Do this using two methods:
  - First find the MAP decoder table using the estimated values of  $\underline{\mathbf{p}}$  and  $\mathbf{Q}$ . Then, construct  $\underline{\hat{\mathbf{x}}}$  from  $\underline{\mathbf{y}}$  according to the decoder table. Finally, compare  $\underline{\hat{\mathbf{x}}}$  and  $\underline{\mathbf{x}}$ .

- (ii) Use the estimated values of  $\underline{\mathbf{p}}$  and  $\mathbf{Q}$  to directly calculate the error probability.
- (h) The error probability when the ML decoder is used. Do this using two methods:
  - (i) First find the ML decoder table using the estimated value of  $\mathbf{Q}$ . Then, construct  $\hat{\underline{\mathbf{x}}}$  from  $\underline{\mathbf{y}}$  according to the decoder table. Finally, compare  $\hat{\underline{\mathbf{x}}}$  and  $\underline{\mathbf{x}}$ .
  - (ii) Use the estimated values of  $\underline{\mathbf{p}}$  and  $\mathbf{Q}$  to directly calculate the error probability.

**Problem 4.** (Optional) Consider a BAC whose  $Q(1|0) = 0.35$  and  $Q(0|1) = 0.55$ . Suppose  $P[X = 0] = 0.4$ .

- (a) Draw the channel diagram.
- (b) Find the joint pmf matrix  $\mathbf{P}$ .
- (c) Find the row vector  $\underline{\mathbf{q}}$  which contains the pmf of the channel output  $Y$ .
- (d) We now analyze the performance of all four reasonable detectors for this binary channel. Complete the table below:

$\hat{x}(y)$	$P$	$\hat{X} = 0   X = 0$	$P$	$\hat{X} = 1   X = 1$	$P(\mathcal{C})$	$P(\mathcal{E})$
$y$						
$1 - y$						
1						
0						

- (e) Find the MAP detector and its error probability.
- (f) Find the ML detector and its error probability.