ECS 452: Digital Communication Systems	**2014/1**

HW 1 — Due: Sep 1

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Instructions

(a) ONE part of a question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work on all of them.

(b) It is important that you try to solve all problems. (5 pt)

(c) Late submission will be heavily penalized.

(d) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Problem 1. Consider the code $\{0, 01\}$

(a) Is it nonsingular?

(b) Is it uniquely decodable?

(c) Is it prefix-free?

Problem 2. Consider the random variable X whose support S_X contains seven values:

$$S_X = \{x_1, x_2, \ldots, x_7\}.$$

Their corresponding probabilities are given by

x	x_1	x_2	x_3	x_4	x_5	x_6	x_7
$p_X(x)$	0.49	0.26	0.12	0.04	0.04	0.03	0.02

(a) Find the entropy $H(X)$.

(b) Find a binary Huffman code for X.

(c) Find the expected codelength for the encoding in part (b).

Problem 3. Find the entropy and the binary Huffman code for the random variable X with pmf

$$p_X(x) = \begin{cases} \frac{x}{21}, & x = 1, 2, \ldots, 6, \\ 0, & \text{otherwise.} \end{cases}$$

Also calculate $\mathbb{E}[\ell(X)]$ when Huffman code is used.

Problem 4. Construct a random variable X (by specifying its pmf) whose corresponding Huffman code is $\{0, 10, 11\}$.

Problem 5. These codes cannot be Huffman codes. Why?

(a) $\{00, 01, 10, 110\}$

(b) $\{01, 10\}$

Hint: Huffman code is optimal.

Problem 6. The following claim is sometimes found in the literature:

> "It can be shown that the length $\ell(x)$ of the Huffman code of a symbol x with probability $p_X(x)$ is always less than or equal to $\lceil -\log_2 p_X(x) \rceil$".

Even though it is correct in many cases, this claim is not true in general.

Find an example where the length $\ell(x)$ of the Huffman code of a symbol x is greater than $\lceil -\log_2 p_X(x) \rceil$.

Hint: Consider a pmf that has the following four probability values $\{0.1, 0.3, 0.34, 0.35\}$.

Problem 7. A memoryless source emits two possible message Y(es) and N(o) with probability 0.9 and 0.1, respectively.

(a) Determine the entropy (per source symbol) of this source.

(b) Find the expected codeword length per symbol of the Huffman binary code for the third-order extensions of this source.

(c) Use `MATLAB` to find the expected codeword length per symbol of the Huffman binary code for the fourth-order extensions of this source.

(d) Use `MATLAB` to plot the expected codeword length per symbol of the Huffman binary code for the nth-order extensions of this source for $n = 1, 2, \ldots, 8$.