

① Suppose $\vec{v} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\vec{u} = \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix}$.

Find (a) $\langle \vec{v}, \vec{u} \rangle$ (b) $\langle \vec{u}, \vec{u} \rangle$ (c) $\|\vec{v}\|$ (d) $\|\vec{u}\|$ (e) $\text{proj}_{\vec{u}} \vec{v}$

$$(a) \langle \vec{v}, \vec{u} \rangle = \vec{u}^H \vec{v} = (-1 \ 3 \ -2) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = -1 + 6 - 6 = -1$$

$$(b) \langle \vec{u}, \vec{u} \rangle = \vec{u}^H \vec{u} = (-1 \ 3 \ -2) \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix} = (-1)^2 + 3^2 + (-2)^2 = 1 + 9 + 4 = 14$$

$$(c) \langle \vec{v}, \vec{v} \rangle = \vec{v}^H \vec{v} = (1 \ 2 \ 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = 1^2 + 2^2 + 3^2 = 1 + 4 + 9 = 14$$

$$\|\vec{v}\| = \sqrt{\langle \vec{v}, \vec{v} \rangle} = \sqrt{14}$$

$$(d) \|\vec{u}\| = \sqrt{\langle \vec{u}, \vec{u} \rangle} = \sqrt{14}$$

$$(e) \text{proj}_{\vec{u}} \vec{v} = \frac{\langle \vec{v}, \vec{u} \rangle}{\langle \vec{u}, \vec{u} \rangle} \vec{u} = \frac{-1}{14} \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix} = \frac{1}{14} \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 1/14 \\ -3/14 \\ 1/7 \end{pmatrix}$$

$$\text{check: } \vec{\sigma} = \vec{v} - \text{proj}_{\vec{u}} \vec{v} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} - \frac{1}{14} \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} = \frac{1}{14} \begin{pmatrix} 14 - 1 \\ 28 + 3 \\ 42 - 2 \end{pmatrix} = \frac{1}{14} \begin{pmatrix} 13 \\ 31 \\ 40 \end{pmatrix}$$

$$\langle \vec{\sigma}, \vec{u} \rangle = \frac{1}{14} (-1 \ 3 \ -2) \begin{pmatrix} 13 \\ 31 \\ 40 \end{pmatrix} = \frac{1}{14} (-13 + 93 - 80) = 0 \quad \checkmark$$

↳ orthogonal!

② Repeat question ① but use

$$\vec{v} = \begin{pmatrix} 1-2j \\ 1+2j \end{pmatrix}, \quad \vec{u} = \begin{pmatrix} 3 \\ 4j \end{pmatrix}.$$

$$(a) \langle \vec{v}, \vec{u} \rangle = \vec{u}^H \vec{v} = (3 \ -4j) \begin{pmatrix} 1-2j \\ 1+2j \end{pmatrix} = 3 - 6j - 4j + 8 = 11 - 10j$$

$$(b) \langle \vec{u}, \vec{u} \rangle = \vec{u}^H \vec{u} = (3 \ -4j) \begin{pmatrix} 3 \\ 4j \end{pmatrix} = 3^2 + 4^2 = 9 + 16 = 25$$

$$(c) \langle \vec{v}, \vec{v} \rangle = \vec{v}^H \vec{v} = (1+2j \ 1-2j) \begin{pmatrix} 1-2j \\ 1+2j \end{pmatrix} = 1^2 + 2^2 + 1^2 + 2^2 = 10$$

$$\|\vec{v}\| = \sqrt{\langle \vec{v}, \vec{v} \rangle} = \sqrt{10}$$

$$(d) \|\vec{u}\| = \sqrt{\langle \vec{u}, \vec{u} \rangle} = \sqrt{25} = 5$$

$$(e) \text{proj}_{\vec{u}} \vec{v} = \frac{\langle \vec{v}, \vec{u} \rangle}{\langle \vec{u}, \vec{u} \rangle} \vec{u} = \frac{11-10j}{25} \begin{pmatrix} 3 \\ 4j \end{pmatrix} = \frac{1}{25} \begin{pmatrix} 33 - 30j \\ 40 + 44j \end{pmatrix}$$