ECS 452: Quiz 4 Solution

Instructions

1. Separate into groups of no more than three persons.
2. The group cannot be the same as your former groups.
3. Only one submission is needed for each group.
4. ***Write down all the steps*** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
5. **Do not panic.**

Name	ID
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For each of the following transition matrices $\mathbf{Q}$, (a) Check whether the corresponding DMC is weakly symmetric and (b) Evaluate the corresponding capacity value.

1. $\mathbf{Q} = \begin{bmatrix} 1/4 & 3/4 \\ 3/4 & 1/4 \end{bmatrix}$ weakly symmetric

$C = \log_2|\mathcal{Y}| - H(\underline{r}) = \log_2 2 - H([\frac{1}{4}\ \frac{3}{4}])$

$= 1 - \left(-\frac{1}{4}\log_2\frac{1}{4} - \frac{3}{4}\log_2\frac{3}{4}\right) \approx 1 - (0.5 + 0.3113)$

$= 1 - 0.8113 = 0.1887$ bpcu is achieved by $p = [1/2\ \ 1/2]$.

2. $\mathbf{Q} = \begin{bmatrix} 0.2 & 0.3 & 0.1 & 0.4 \\ 0.3 & 0.2 & 0.4 & 0.1 \end{bmatrix}$ weakly symmetric

$C = \log_2|\mathcal{Y}| - H(\underline{r}) = \log_2 4 - H([0.2\ 0.3\ 0.1\ 0.4])$

$\approx 2 - (\underbrace{-0.1\log_2 0.1}_{0.3322} \underbrace{-0.2\log_2 0.2}_{0.4644} \underbrace{-0.3\log_2 0.3}_{0.5211} \underbrace{-0.4\log_2 0.4}_{0.5288})$

$= 2 - 1.8464 = 0.1536$ bpcu is achieved by $p = [1/2\ \ 1/2]$.

3. $\mathbf{Q} = \begin{bmatrix} 0 & 0.2 & 0 & 0.4 & 0 & 0 & 0.4 \\ 0.1 & 0 & 0.9 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.3 & 0.7 & 0 \end{bmatrix}$ Not weakly symmetric (fail both (1) and (2) in the definition)

Note that there is only one non-zero element in each column. So, the channel has NO^2.

$C = \log_2|\mathcal{X}| = \log_2 3 \approx 1.5850$ bpcu is achieved by $p = [1/3\ \ 1/3\ \ 1/3]$.

4. $\mathbf{Q} = \begin{bmatrix} 1/6 & 5/6 \\ 1/6 & 5/6 \end{bmatrix}$ Not weakly symmetric (fail (2) in the definition)

Note that the rows are all exactly the same. So, $Q(y|x)$ does not depend on x. This implies $X \perp\!\!\!\perp Y$. Therefore, $I(X;Y) = 0$ regardless of p. So, $C = 0$ bpcu is obtained by any p.