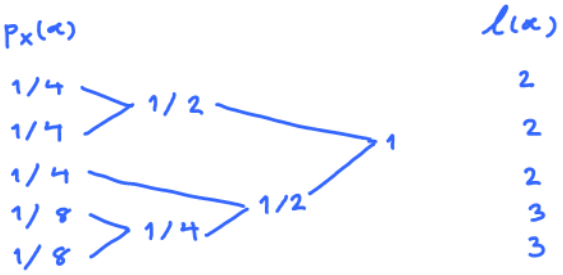
ECS 452: Quiz 1

Instructions

1. Separate into groups of no more than three persons.
2. Only one submission is needed for each group.
3. ***Write down all the steps*** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
4. **Do not panic.**

Name	ID

1. Consider a random variable X which has five possible values. Their probabilities are 1/4, 1/4, 1/4, 1/8, 1/8.
   a. Find the expected codeword length when Huffman coding is used without extension.



$$\mathbb{E}[\ell(X)] = 3\times 2\times\tfrac{1}{4} + 2\times 3\times\tfrac{1}{8} = \tfrac{6}{4}+\tfrac{3}{4} = \tfrac{9}{4} = 2.25 \text{ bits/symbol}$$

   b. Find the entropy (per symbol) of this random variable.

$$H(X) = \sum_a p_X(a)\log_2 p_X(a) = -3\times\tfrac{1}{4}\log_2\tfrac{1}{4} - 2\times\tfrac{1}{8}\log_2\tfrac{1}{8}$$

$$= 3\times\tfrac{2}{4} + 2\times\tfrac{3}{8} = \tfrac{9}{4} = 2.25 \text{ bits/symbol}$$

2. No need to provide any explanation for this question.
   Consider a DMC whose samples of input and output are provided below

```
x: 1  1  1  0  1  0  1  0  1  1  1  1  1  1  1
y: 1  1  1  0  1  0  1  1  1  1  1  1  1  1  1
```

   Estimate the following quantities:

   a. $P[X=0] \approx 3/15 = 1/5 = 0.2$

   b. $p(1) \equiv P[X=1] \approx 12/15 = 4/5 = 0.8$

   c. $p_Y(0) \equiv P[Y=0] \approx 2/15 \approx 0.133$

   d. $q(1) \equiv P[Y=1] \approx 13/15 \approx 0.867$

   e. $P[Y=0 \mid X=0] \approx \frac{2}{3} \approx 0.667$

   f. $p_{Y|X}(1|0) \equiv P[Y=1 \mid X=0] \approx \frac{1}{3} \approx 0.333$

   g. $Q(0|1) \equiv [Y=0 \mid X=1] \approx \frac{0}{15} = 0$

   h. $Q(1|1) \equiv P[Y=1 \mid X=1] \approx \frac{15}{15} = 1$

   i. Matrix Q $\approx$

$$\begin{array}{c|cc} x \backslash y & 0 & 1 \\ 0 & 2/3 & 1/3 \\ 1 & 0 & 1 \end{array}$$

   j. $P[X=0, Y=0] \approx \frac{2}{15}$ ← Note that this is the same as

$$P[Y=0 \mid X=0]\,P[X=0]$$

$$\frac{2}{3}\times\frac{1}{5} = \frac{2}{15}$$