**ECS 452: Digital Communication Systems** **2013/1**

# HW 4 — Due: Oct 4

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**Instructions**

(a) ONE part of a question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work on all of them.

(b) It is important that you try to solve all problems. (5 pt)

(c) Late submission will be heavily penalized.

(d) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

**Problem 1.** Consider random variables $X$ and $Y$ whose joint pmf is given by

$$p_{X,Y}(x,y) = \begin{cases} c(x+y), & x \in \{1,3\} \text{ and } y \in \{2,4\}, \\ 0, & \text{otherwise.} \end{cases}$$

Evaluate the following quantities.

(a) $H(X,Y)$

(b) $H(X)$

(c) $H(Y)$

(d) $H(X|Y)$

(e) $H(Y|X)$

(f) $I(X;Y)$

**Problem 2.** Consider a pair of random variables $X$ and $Y$ whose joint pmf is given by

$$p_{X,Y}(x,y) = \begin{cases} 1/15, & x=3, y=1, \\ 2/15, & x=4, y=1, \\ 4/15, & x=3, y=3, \\ \beta, & x=4, y=3, \\ 0, & \text{otherwise.} \end{cases}$$

Evaluate the following quantities.

(a) $H(X,Y)$

(b) $H(X)$

(c) $H(Y)$

(d) $H(X|Y)$

(e) $H(Y|X)$

(f) $I(X;Y)$

**Problem 3.** Compute the capacities of each of the communication channels whose transition probability matrices are specified below.

(a)
$$Q = \begin{bmatrix} 1/3 & 2/3 \\ 2/3 & 1/3 \end{bmatrix}$$

(b)
$$Q = \begin{bmatrix} 0 & 1/5 & 4/5 & 0 & 0 \\ 2/3 & 0 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 0 & 1. \end{bmatrix}$$

(c)
$$Q = \begin{bmatrix} 1/3 & 2/3 & 0 \\ 0 & 2/3 & 1/3. \end{bmatrix}$$

(d)
$$Q = \begin{bmatrix} 1/3 & 2/3 \\ 1/3 & 2/3 \end{bmatrix}$$

**Problem 4** (Blahut-Arimoto algorithm)**.**

(a) Create a `MATLAB` function `capacity` which calculates the capacity $C = \max_p I(p,Q)$ and the corresponding capacity-achieving input pmf $p^*$ using Blahut-Arimoto algorithm.

The function takes two inputs: (1) the channel transition probability matrix $Q(y|x)$ and (2) the initial guess of the pmf $p_0(x)$.

Define a sequence $p_r(x)$, $r = 0, 1, \dots$ according to the following iterative prescription

$$p_{r+1}(x) = \frac{p_r(x)\, c_r(x)}{\sum_x p_r(x)\, c_r(x)},$$

where

$$\log c_r(x) = \sum_y Q(y|x) \log \frac{Q(y|x)}{q_r(y)} \tag{4.1}$$

and

$$q_r(y) = \sum_x p_r(x) Q(y|x).$$

After several iterations, the pmf $p_r(x)$ will converge to the capacity-achieving one. In fact,

$$\log\left(\sum_x p_r(x) c_r(x)\right) \le C \le \log\left(\max_x c_r(x)\right). \tag{4.2}$$

So, we can use (4.2) to control the accuracy of our results.

(b) Check your answers in Problem 3 using the Blahut-Arimoto algorithm.