ECS 452: Digital Communication Systems **2013/1**

HW 2 — Due: Not Due

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Problem 1. In a binary antipodal signaling scheme, the message S is randomly selected from the alphabet set $\mathcal{S} = \{3, -3\}$ with $p_1 = P[S = -3] = 0.3$ and $p_2 = P[S = 3] = 0.7$. The message is corrupted by an independent additive exponential noise N whose pdf is

$$f_N(n) = \begin{cases} \frac{1}{2}e^{-n/2}, & n \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the MAP detector $\hat{s}_{\text{MAP}}(r)$.

(b) Indicate the decision regions of the MAP detector in part (a).

(c) Consider a detector of the form

$$\hat{s}(r) = \begin{cases} 3, & r > \tau, \\ -3, & r \leq \tau \end{cases}$$

for some threshold τ. Find and then plot the probability of (symbol detection) error for this detector as a function of τ. Hint: The plots from actual simulation are shown in class. The same plots are shown in Figure 2.1.

(d) Evaluate the error probability of the MAP detector.

(e) Evaluate the error probability of the ML detector.

Problem 2. Repeat parts (a)-(d) of Question 1 but now the noise is uniform on $[-4, 4]$.

Problem 3. In a ternary signaling scheme, the message S is randomly selected from the alphabet set $\mathcal{S} = \{-1, 1, 4\}$ with $p_1 = P[S = -1] = 0.3 = p_2 = P[S = 1]$ and $p_3 = P[S = 4] = 0.4$. The message is corrupted by an independent additive Gaussian noise $N \sim \mathcal{N}(0, 2)$.

(a) Find the average signal energy[1] E_s.

Note that

$$E_s = \sum_i p_i |s_i|^2.$$

[1] Same as "average symbol energy" or "average energy per symbol" or "average energy per signal"

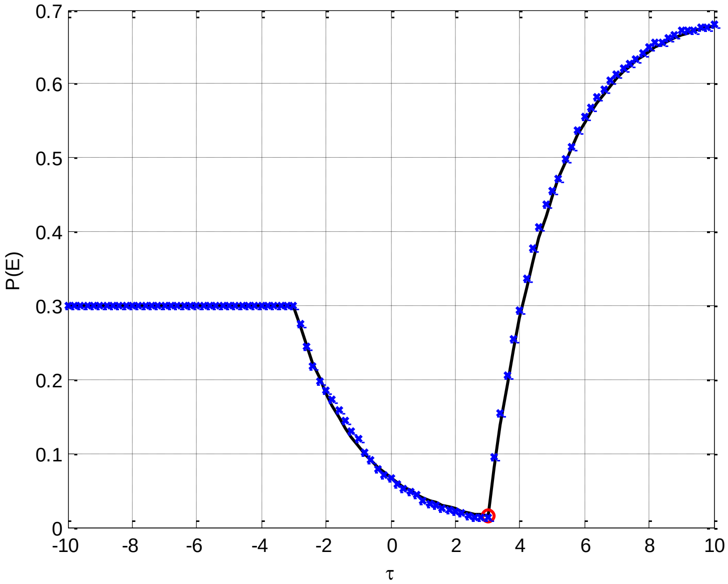

Figure 2.1: $P(\mathcal{E})$ for Exponential Noise in Question 1

(b) Find the MAP detector $\hat{s}_{\text{MAP}}(r)$.

(c) Indicate the decision regions of the MAP detector in part (b).

(d) Evaluate the error probability of the MAP detector.

Problem 4. In a ternary signaling scheme, the message S is randomly selected from the alphabet set $\mathcal{S} = \{-1, 1, 4\}$ with $p_1 = P[S = -1] = 0.41$, $p_2 = P[S = 1] = 0.08$ and $p_3 = P[S = 4] = 0.51$. The message is corrupted by an independent additive Gaussian noise $N \sim \mathcal{N}(0, 2)$.

(a) Find the average signal energy E_s.

(b) If the MAP detector is used, find $P(\mathcal{E}|S = 1)$; that is, find the probability of (decoding) error given that $S = 1$ was transmitted.

Problem 5. In a **standard** quaternary signaling scheme, the message S is equiprobably selected from the alphabet set $\mathcal{S} = \left\{-\frac{3d}{2}, -\frac{d}{2}, \frac{d}{2}, \frac{3d}{2}\right\}$. The message is corrupted by an independent additive exponential noise N whose pdf is

$$f_N(n) = \begin{cases} \lambda e^{-\lambda n}, & n \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the average symbol energy.

(b) Find the average energy per bit.

(c) Find the MAP detector $\hat{s}_{\text{MAP}}(r)$.

(d) Evaluate the error probability of the MAP detector.

(e) Let $\lambda = \frac{1}{\sigma}$. (This is to set $\operatorname{Var} N = \sigma^2$ as in the case for Gaussian noise.) Plot $\frac{E_b}{\sigma^2}$ vs. probability of error $P(\mathcal{E})$. Consider $\frac{E_b}{\sigma^2}$ from -30 to 10 dB.