ECS 452: Digital Communication Systems **2013/1**

HW 1 — Due: July 24

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Instructions

(a) ONE part of a question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work on all of them.

(b) It is important that you try to solve all problems. (5 pt)

(c) Late submission will be heavily penalized.

(d) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Problem 1. Consider the **two-path channels** in which the receive signal is given by

$$y(t) = \beta_1 x(t - \Delta t_1) + \beta_2 x(t - \Delta t_2).$$

Four different cases are considered.

(a) Small $|\Delta t_1 - \Delta t_2|$ and $|\beta_1| \gg |\beta_2|$

(b) Large $|\Delta t_1 - \Delta t_2|$ and $|\beta_1| \gg |\beta_2|$

(c) Small $|\Delta t_1 - \Delta t_2|$ and $|\beta_1| \approx |\beta_2|$

(d) Large $|\Delta t_1 - \Delta t_2|$ and $|\beta_1| \approx |\beta_2|$

Figure 1.1 shows four plots of normalized[1] $|X(f)|$ (dotted black line) and normalized $|Y(f)|$ (blue line) in [dB]. Match the four graphs (i-iv) to the four cases (a-d).

Problem 2. Consider four vectors

$$\boldsymbol{v}^{(1)} = \begin{pmatrix} 1+j \\ 1-j \\ 0 \end{pmatrix}, \boldsymbol{v}^{(2)} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \boldsymbol{v}^{(3)} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \text{ and } \boldsymbol{v}^{(4)} = \begin{pmatrix} -1 \\ -1 \\ -j \end{pmatrix}.$$

[1] The function is normalized so that the maximum point is ~~1~~ 0 dB.

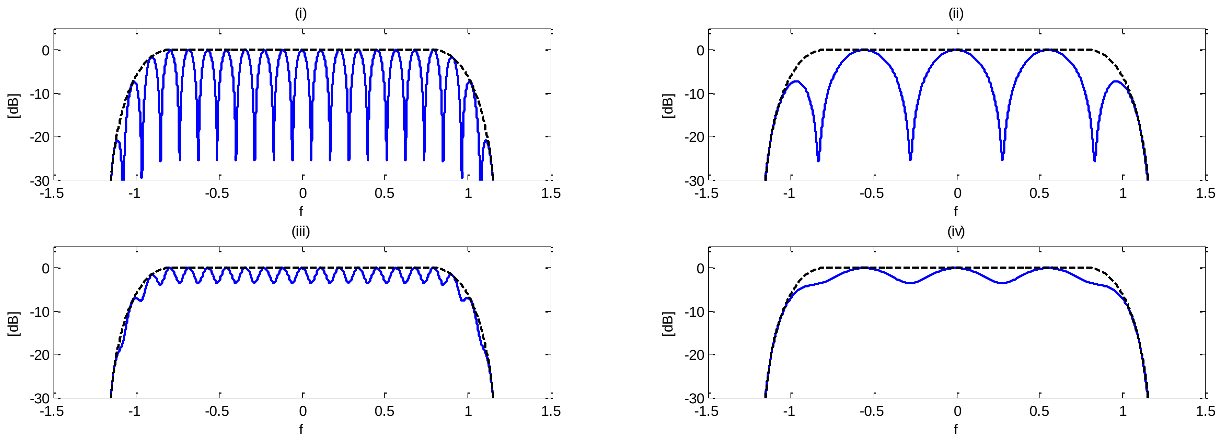

Figure 1.1: Frequency selectivity in the receive spectra (blue line) for two-path channels.

(a) Use the Gram-Schmidt orthogonalization procedure (GSOP) (where the vectors are applied in the order given) to find the orthonormal vectors $\boldsymbol{e}^{(1)}$, $\boldsymbol{e}^{(2)}$, … that can be used to represent $\boldsymbol{v}^{(1)}$, $\boldsymbol{v}^{(2)}$, $\boldsymbol{v}^{(3)}$, and $\boldsymbol{v}^{(4)}$.

(b) Use the orthonormal vectors from the previous part to construct the matrix $E = \left[\boldsymbol{e}^{(1)}\ \boldsymbol{e}^{(2)}\ \cdots\right]$. Find the matri~~c~~x C such that $V = EC$ where $V = \left[\boldsymbol{v}^{(1)}\ \boldsymbol{v}^{(2)}\ \boldsymbol{v}^{(3)}\ \boldsymbol{v}^{(4)}\right]$.

Hint: Implement GSOP in MATLAB.

Problem 3. Consider the two signals $s_1(t)$ and $s_2(t)$ shown in Figure 1.2. Note that V and T_b are some positive constants. Your answers should be given in terms of them.

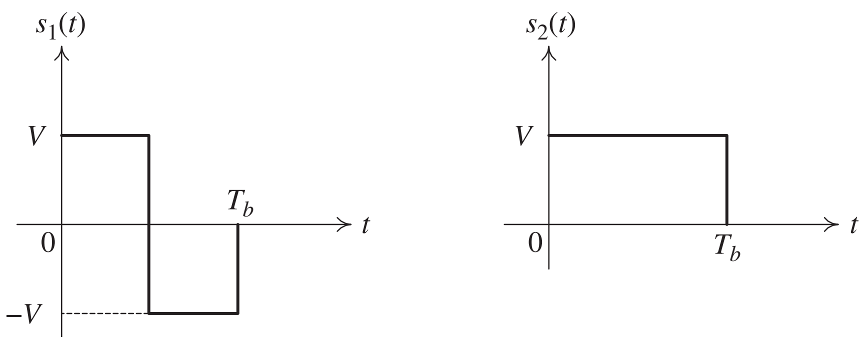

Figure 1.2: Signal set for Question 3

(a) Find the energy in each signal.

(b) Use the Gram-Schmidt orthogonalization procedure (GSOP) (where the signals are applied in the order given) to find two orthonormal functions $\phi_1(t)$ and $\phi_2(t)$ that can be used to represent $s_1(t)$ and $s_2(t)$.

(c) Find the two vectors that represent the two waveforms $s_1(t)$ and $s_2(t)$ in the new (signal) space based on the orthonormal basis found in the previous part. Draw the corresponding constellation.

Problem 4. Consider the two signals $s_1(t)$ and $s_2(t)$ shown in Figure 1.3. Note that V, α and T_b are some positive constants.

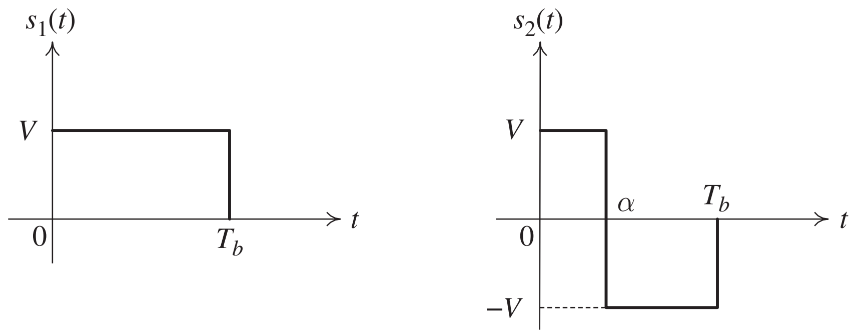

Figure 1.3: Signal set for Question 4

(a) Find the energy in each signal.

(b) Use the Gram-Schmidt orthogonalization procedure (GSOP) (where the signals are applied in the order given) to find two orthonormal functions $\phi_1(t)$ and $\phi_2(t)$ that can be used to represent $s_1(t)$ and $s_2(t)$.

(c) Plot $\phi_1(t)$ and $\phi_2(t)$ when $\alpha = \frac{T_b}{4}$.

(d) Find the two vectors $\boldsymbol{s}^{(1)}$ and $\boldsymbol{s}^{(2)}$ that represent the two waveforms $s_1(t)$ and $s_2(t)$ in the new (signal) space based on the orthonormal basis found in the previous part.

(e) Draw the corresponding constellation when $\alpha = \frac{T_b}{4}$.

(f) Draw $\boldsymbol{s}^{(2)}$ when $\alpha = \frac{k}{10}T_b$ where $k = 1, 2, \ldots, 9$.