Sirindhorn International Institute of Technology

Thammasat University

School of Information, Computer and Communication Technology

ECS452 2013/1 Part I.1 Dr.Prapun

1 Elements of a Digital Communication System

1.1. Figure 1 illustrates the functional diagram and the basic elements of a digital communication system.

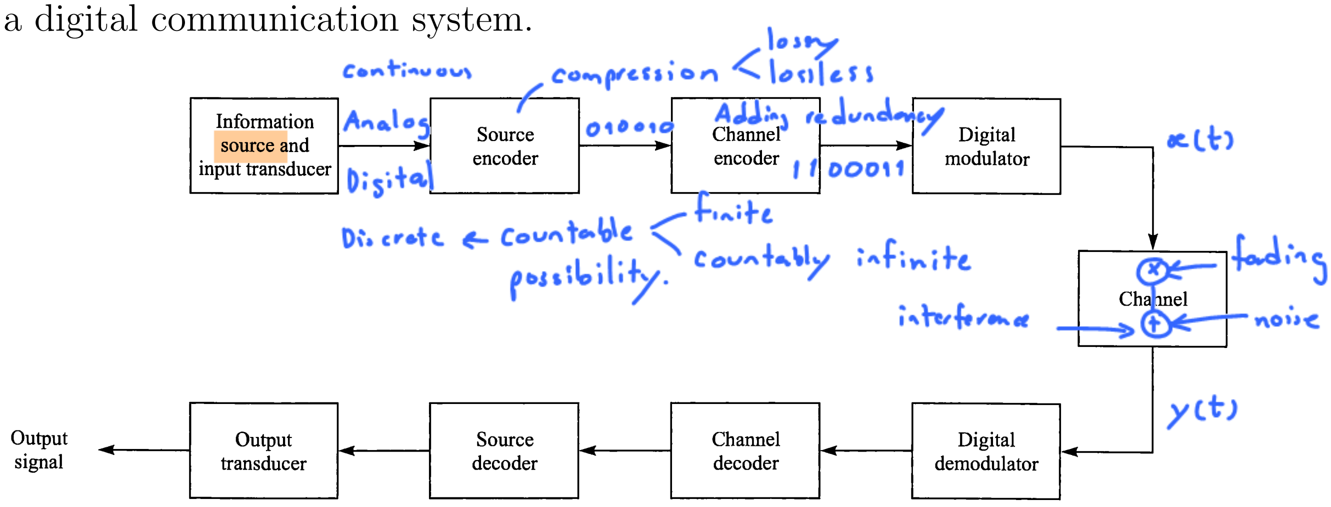


Figure 1: Basic elements of a digital communication system

1.2. The source output may be either

- an **analog** signal, such as an audio or video signal,

or

0 or 1

4 possible symbols

- a **digital** signal, such as the output of a computer, that is discrete in time and has a finite number of output characters.

1.3. Source Coding: The messages produced by the source are converted into a sequence of binary digits.

- Ideally, we would like to represent the source output (message) by as few binary digits as possible.
- In other words, we seek an efficient representation of the source output that results in little or no redundancy.
- The process of efficiently converting the output of either an analog or digital source into a sequence of binary digits is called **source encoding** or **data compression**.

1.4. Channel Coding: The sequence of binary digits from the source encoder, which we call the information sequence, is passed to the channel encoder.

- The purpose of the channel encoder is to introduce, in a controlled manner, some *redundancy* in the binary information sequence that can be used at the receiver to overcome the effects of noise and interference encountered in the transmission of the signal through the channel.
- Thus, the added redundancy serves to increase the reliability of the received data and improves the fidelity of the received signal.
- See Examples 1.5 and 1.6.

Example 1.5. Trivial channel coding: Repeat each binary digit m times, where m is some positive integer.

Example 1.6. More sophisticated channel coding: Taking k information bits at a time and mapping each k-bit sequence into a unique n-bit sequence, called a **code word**.

- The amount of redundancy introduced by encoding the data in this manner is measured by the ratio n/k. The reciprocal of this ratio, namely k/n, is called the rate of the code or, simply, the **code rate**.

1.7. The binary sequence at the output of the channel encoder is passed to the **digital modulator**, which serves as the interface to the communication channel.

physical

- Since nearly all the communication channels encountered in practice are capable of transmitting electrical signals (waveforms), the primary purpose of the digital modulator is to map the binary information sequence into signal waveforms.

- The digital modulator may simply map the binary digit 0 into a waveform $s_0(t)$ and the binary digit 1 into a waveform $s_1(t)$. In this manner, each bit from the channel encoder is transmitted separately.
We call this **binary modulation**.

- The modulator may transmit b coded information bits at a time by using $M = 2^b$ distinct waveforms $s_i(t), i = 0, 1, \ldots, M - 1$, one waveform for each of the 2^b possible b-bit sequences.
We call this ***M*-ary modulation** $(M > 2)$.

1.8. The **communication channel** is the physical medium that is used to send the signal from the transmitter to the receiver.

- The physical channel may be
  - a pair of wires that carry the electrical signal, or
  - an optical fiber that carries the information on a modulated light beam, or
  - an underwater ocean channel in which the information is transmitted acoustically, or
  - free space over which the information-bearing signal is radiated by use of an antenna.

  Other media that can be characterized as communication channels are data storage media, such as magnetic tape, magnetic disks, and optical disks.

- Whatever the physical medium used for transmission of the information, the essential feature is that the transmitted signal is corrupted in a random manner by a variety of possible mechanisms, such as additive **thermal noise** generated by electronic devices; man-made noise, e.g., automobile ignition noise; and **atmospheric noise**, e.g., electrical lightning discharges during thunderstorms.

- Other channel impairments including noise, attenuation, distortion, fading, and interference (such as interference from other users of the channel).

1.9. At the receiving end of a digital communication system, the digital demodulator processes the channel-corrupted transmitted waveform and reduces the waveforms to a sequence of numbers that represent estimates of the transmitted data symbols (binary or M-ary).

This sequence of numbers is passed to the channel decoder, which attempts to reconstruct the original information sequence from knowledge of the code used by the channel encoder and the redundancy contained in the received data.

- A measure of how well the demodulator and decoder perform is the frequency with which errors occur in the decoded sequence. More precisely, the average probability of a bit-error at the output of the decoder is a measure of the performance of the demodulator-decoder combination.

- In general, the probability of error is a function of the code characteristics, the types of waveforms used to transmit the information over the channel, the transmitter power, the characteristics of the channel (i.e., the amount of noise, the nature of the interference), and the method of demodulation and decoding.

1.10. As a final step, when an analog output is desired, the source decoder accepts the output sequence from the channel decoder and, from knowledge of the source encoding method used, attempts to reconstruct the original signal from the source.

- Because of channel decoding errors and possible distortion introduced by the source encoder, and perhaps, the source decoder, the signal at the output of the source decoder is an approximation to the original source output.

- The difference or some function of the difference between the original signal and the reconstructed signal is a measure of the distortion introduced by the digital communication system.

2 Mathematical Models for Communication Channels

In the design of communication systems for transmitting information through physical channels, we find it convenient to construct mathematical models that reflect the most important characteristics of the transmission medium. Then, the mathematical model for the channel is used in the design of the channel encoder and modulator at the transmitter and the demodulator and channel decoder at the receiver.

2.1. Here, we provide a brief description of the channel models that are frequently used to characterize many of the physical channels that we encounter in practice.

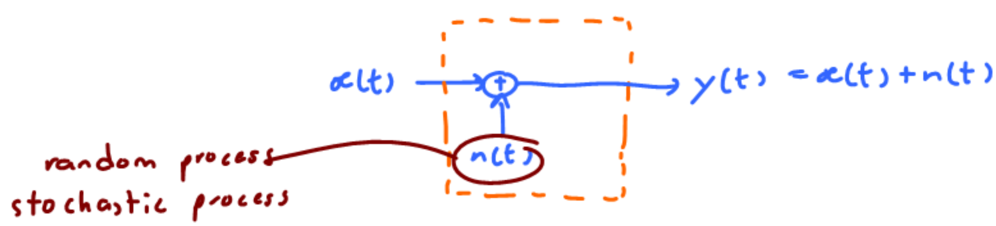
(a) Additive Noise Channel

(b) Linear Filter Channel

(c) Linear Time-Variant Filter Channel

(a) **2.2. Additive Noise Channel**

- In this model, the transmitted signal $x(t)$ is corrupted by an additive random noise process $n(t)$.



- Physically, the additive noise process may arise from electronic components and amplifiers at the receiver of the communication system or from interference encountered in transmission (as in the case of radio signal transmission).

- If the noise is introduced primarily by electronic components and amplifiers at the receiver, it may be characterized as thermal noise. This type of noise is characterized statistically as a Gaussian noise process.

  Hence, the resulting mathematical model for the channel is usually called the **additive Gaussian noise channel**.

- Because this channel model applies to a broad class of physical communication channels and because of its mathematical tractability, this is the predominant channel model used in our communication system analysis and design.

- Channel attenuation is easily incorporated into the model. When the signal undergoes attenuation in transmission through the channel, the received signal is

$$y(t) = \beta x(t) + n(t)$$

$$g(t) * \delta(t-\mu) = g(t-\mu)$$

where β is the attenuation factor.

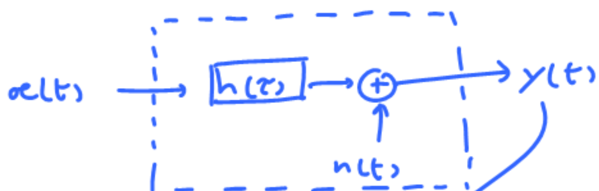
(b) 2.3. Linear Filter Channel

Example

$$y(t) = \sum_i \beta_i x(t - \Delta t_i) + n(t)$$

$$= x(t) * h(t) + n(t)$$

$$h(t) = \sum_i \beta_i \delta(t - \Delta t_i)$$



$$y(t) = x(t) * h(t) + n(t) = \int_{-\infty}^{\infty} h(\tau) x(t-\tau)\, d\tau + n(t)$$

- See 2.5 to learn more about multipath propagation.

(c) 2.4. Linear Time-Variant Filter Channel or Linear Time-Varying Channel

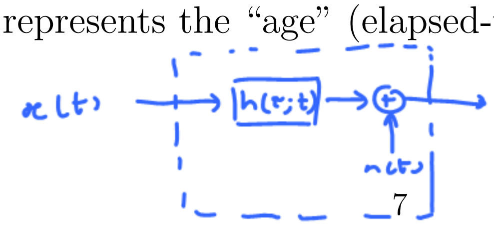
- Physical channels such as radio channels result in time-variant multipath propagation of the transmitted signal.

- The channels may be characterized mathematically as time-variant linear filters.

- Such linear filters are characterized by a time-variant channel impulse response $h(\tau;t)$, where $h(\tau;t)$ is the response of the channel at time t due to an impulse applied at time $t-\tau$.

- Thus, τ represents the "age" (elapsed-time) variable.

replace $h(\tau)$ with $h(\tau;t)$.



$$y(t) = \int_{-\infty}^{\infty} h(\tau;t) x(t-\tau)\, d\tau + n(t)$$

- See Example 2.7.

2.5. Multipath Propagation and Time Dispersion [2, p 1]

- The presence of multiple scatterers (buildings, vehicles, hills, and so on) causes a transmitted radio wave to propagate along several different paths that terminate at the receiver. Hence, the receive antenna picks up a superposition of multiple attenuated copies of the transmit signal.

  This phenomenon is referred to as **multipath propagation**. [2, p 1]

- Due to different lengths of the propagation paths, the individual multipath components experience different delays (time shifts) [2, p 1]:

$$y(t) = x(t) * h(t) + n(t) = \sum_{i=0}^{v} \beta_i x(t - \Delta t_i) + n(t)$$

  where

$$h(t) = \sum_{i=0}^{v} \beta_i \delta(t - \Delta t_i).$$

  Here, $\beta_i = |\beta_i| e^{j\phi_i}$ and Δt_i are, respectively, the complex attenuation factor and delay associated with the ith path.

  The corresponding frequency response of the channel is

$$H(f) = \sum_{i=0}^{v} \beta_i e^{-j2\pi f \Delta t_i}. \qquad (1)$$

  - See Ex. 2.6.

- The receiver thus observes a temporally smeared-out version of the transmit signal. Even though the medium itself is not physically dispersive (in the sense that different frequencies propagate with different velocities), such channels are termed **time-dispersive**.

- Time-dispersive channels are **frequency-selective** in the sense that different frequencies are attenuated differently. This is clear from the f-dependent $H(f)$ in (1).

  - These differences in attenuation become more severe when the difference of the path delays is large and the difference between the path attenuations is small. [2, p 3]

* This can be seen from the expression in Ex. 2.6. See also Figure 1.1 in [2].

- Although multipath propagation has traditionally been viewed as a transmission impairment, nowadays there is a tendency to consider it as beneficial since it provides additional degrees of freedom that are known as delay diversity or frequency diversity and that can be exploited to realize diversity gains or, in the context of multiantenna systems, even multiplexing gains. [2]

Example 2.6. Two-path channel: Consider two propagation paths in a static environment. The receive signal in the equivalent complex baseband domain is given by

$$y(t) = \beta_1 x(t - \Delta t_1) + \beta_2 x(t - \Delta t_2).$$

$z + z^* = 2\mathrm{Re}\{z\} = 2\mathrm{Re}\{z^*\}$

In which case, $h(t) = \beta_1 \delta(t - \Delta t_1) + \beta_2 \delta_2 \cancel{x}(t - \Delta t_2)$ and

$|z|^2 = zz^*$

$$H(f) = \beta_1 e^{-j2\pi f \Delta t_1} + \beta_2 e^{-j2\pi f \Delta t_2}$$

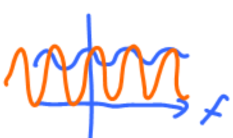
$|z_1 + z_2|^2 = (z_1 + z_2)(z_1^* + z_2^*)$

and

$$\begin{aligned} H(f) &= \beta_1 e^{-j2\pi f \Delta t_1} + \beta_2 e^{-j2\pi f \Delta t_2} \\ &= |\beta_1| e^{j\phi_1} e^{-j2\pi f \Delta t_1} + |\beta_2| e^{j\phi_2} e^{-j2\pi f \Delta t_2} \end{aligned}$$

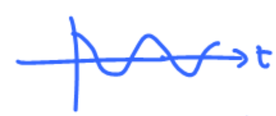
$z_1 z_2^* + z_1^* z_2$

Recall that $|Z_1 + Z_2|^2 = |Z_1|^2 + |Z_2|^2 + 2\mathrm{Re}\{Z_1 Z_2^*\}$. Therefore,



$$\begin{aligned} |H(f)|^2 &= |\beta_1|^2 + |\beta_2|^2 + 2\,\mathrm{Re}\left\{ |\beta_1| e^{j\phi_1} e^{-j2\pi f \Delta t_1} \left(|\beta_2| e^{j\phi_2} e^{-j2\pi f \Delta t_2} \right)^* \right\} \\ &= |\beta_1|^2 + |\beta_2|^2 + 2|\beta_1||\beta_2| \cos\left(2\pi(\Delta t_2 - \Delta t_1) f + (\phi_1 - \phi_2)\right). \end{aligned}$$

$x(t) = \cos(2\pi 5 t)$



Example 2.7. A Linear Time-Variant Filter Channel:

$$y(t) = \sum_{k=1}^{L} \beta_k(t)\, x(t - \Delta t_k) + n(t).$$

$x(f) = \cos(2\pi 5 f)$

Here, the received signal consists of L multipath components, where the kth component is attenuated by $\beta_k(t)$ and delayed by Δt_k. The time-variant impulse response has the form

$$h(\tau; t) = \sum_{k=1}^{L} \beta_k(t)\, \delta(\tau - \Delta t_k).$$

3 PAM: A Digital Modulation Scheme

3.1. The mapping between the digital sequence (which we assume to be a binary sequence) and the signal sequence to be transmitted over the channel can be either memoryless or with memory, resulting in memoryless modulation schemes and modulation schemes with memory.

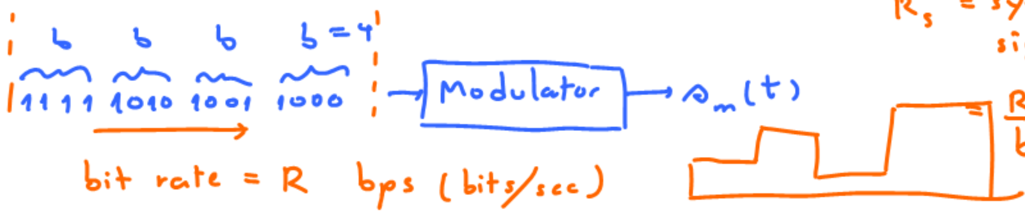
Definition 3.2. In a **memoryless modulation** scheme, the binary sequence is parsed into subsequences each of length b, and each sequence is mapped into one of the $s_m(t)$, $1 \le m \le 2^b$, signals regardless of the previously transmitted signals.

M

$b = \log_2 M$

- This modulation scheme is equivalent to a mapping from $M = 2^b$ messages to M possible signals.

R_s = symbol rate
signaling rate



$\frac{R}{b} = \frac{R}{\log_2 M}$

The waveforms $s_m(t)$ used to transmit information over the communication channel can be, in general, of any form. However, usually these waveforms are bandpass signals which may differ in amplitude or phase or frequency, or some combination of two or more signal parameters.

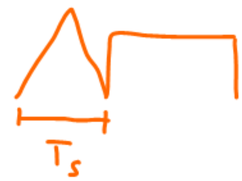
Definition 3.3. In a modulation scheme with memory, the mapping is from the set of the current b bits and the past $(L-1)b$ bits to the set of possible $M = 2^b$ messages.

- Modulation systems with memory are effectively represented by Markov chains.
- The transmitted signal depends on the current b bits as well as the most recent $L-1$ blocks of b bits.
- This defines a finite-state machine with $2^{(L-l)b}$ states.
- The mapping that defines the modulation scheme can be viewed as a mapping from the current state and the current input of the modulator to the set of output signals resulting in a new state of the modulator.
- Parameter L is called the **constraint length** of modulation.

- The case of $L = 1$ corresponds to a memoryless modulation scheme.

Definition 3.4. We assume that these signals (selected from $\{s_m(t)\}$ are transmitted at every T_s seconds, where T_s is called the **signaling interval**.

- This means that in each second



$$R_s = \frac{1}{T_s}$$

symbols are transmitted.

Parameter R_s is called the **signaling rate** or **symbol rate**.

$R_s = \frac{R}{b}$

3.5. If the energy content of $s_m(t)$ is denoted by $\mathcal{E}_m$, then the **average signal energy** is given by

Given a signal $x(t)$,

$$\mathcal{E}_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

average energy per symbol $\rightarrow$
$$\mathcal{E}_{\text{avg}} = \sum_{m=1}^{M \leftarrow 16} p_m \mathcal{E}_m$$

where P_m indicates the probability of the mth signal (message probability).

- For equiprobable signals,

$$p_m = \frac{1}{M} \Rightarrow \mathcal{E}_{avg} = \frac{1}{M} \sum_{m=1}^{M} \mathcal{E}_m$$

average energy per bit

$$\mathcal{E}_{bavg} = \frac{\mathcal{E}_{avg}}{b} = \frac{\mathcal{E}_{avg}}{\log_2 M}$$

average power

$$P_{avg} = \frac{\mathcal{E}_{avg}}{T_s} = R_s \mathcal{E}_{avg}$$

$$= \frac{\mathcal{E}_{bavg}}{(1/R)} = R \mathcal{E}_{bavg}$$

- If all signals have the same energy, then
  - $\mathcal{E}_m \equiv \mathcal{E}$ for some $\mathcal{E}$ and
  - $\mathcal{E}_{\text{avg}} = \mathcal{E}$.

3.6. Pulse Amplitude Modulation (PAM): The signal waveforms may be represented as

$$s_m(t) = A_m p(t), \quad 1 \le m \le M$$

where $p(t)$ is a pulse and $\{A_m, 1 \le m \le M\}$ denotes the set of M possible amplitudes corresponding to $M = 2^b$ possible b-bit blocks of symbols.

- Usually, the signal amplitudes A_m take the discrete values

$$A_m = 2m - 1 - M, \quad m = 1, 2, \ldots, M$$

$M=2 \quad \mathcal{A} = \{\pm 1\}$

i.e., the amplitudes are $\{\pm 1, \pm 3, \pm 5, \ldots, \pm(M-1)\} = \mathcal{A}$ $\quad M=4 \quad \mathcal{A} = \{\pm 1, \pm 3\}$

- The shape of $p(t)$ influences the spectrum of the transmitted signal.
- The energy in signal $s_m(t)$ is given by

$$\mathcal{E}_m = \int_{-\infty}^{\infty} |s_m(t)|^2 \, dt = \int_{-\infty}^{\infty} |A_m p(t)|^2 dt = A_m^2 \underbrace{\int_{-\infty}^{\infty} |p(t)|^2 dt}_{\mathcal{E}_p}$$

$$\mathcal{E}_m = A_m^2 \mathcal{E}_p$$

- For equiprobable signals,

$$\mathcal{E}_{avg} = \sum_{m=1}^{M} \underbrace{p_m}_{\frac{1}{M}} \mathcal{E}_m = \frac{1}{M} \sum_{m=1}^{M} \mathcal{E}_m = \frac{1}{M} \sum_{m=1}^{M} A_m^2 \mathcal{E}_p = \frac{\mathcal{E}_p}{M} \sum_{m=1}^{M} A_m^2$$

$$= \frac{\mathcal{E}_p}{M} \left(1^2 + (-1)^2 + 3^2 + (-3)^2 + 5^2 + (-5)^2 + \cdots + (-(M-1))^2 + (M-1)^2 \right)$$

$$= \frac{2\mathcal{E}_p}{M} \left(1^2 + 3^2 + 5^2 + \cdots + (M-1)^2 \right) = \; ? \quad \text{(HW)}$$

Ex. $M = 2$

$$\mathcal{E}_{avg} = \frac{2\mathcal{E}_p}{2}(1^2) = \mathcal{E}_p$$

Ex. $M = 4$

$$\mathcal{E}_{avg} = \frac{2\mathcal{E}_p}{4}(1^2 + 3^2) = 5\mathcal{E}_p$$