ECS 452: In-Class Exercise # 7 Sol

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups.** Only one submission is needed for each group.
2. **[ENRE] Explanation is not required for this exercise.**
3. **[WASP] Write your answer(s) in the corresponding space(s) provided.**
4. **Do not panic.**

Date: 7 / 2 / 2020			
Name	ID (last 3 digits)		

1. Consider a binary channel whose $Q(0|0) = 0.7$ and $Q(0|1) = 0.3$. Suppose $P[X = 1] = 0.4$.
   Find the channel matrix **Q**, the output probability vector $\underline{\mathbf{q}}$, and the joint pmf matrix **P**.

$P[X = 0] = 1 - 0.4 = 0.6$

The question explicitly specifies two elements in the **Q** matrix.

The other two elements can be calculated by using the fact that, in the **Q** matrix, any row sum should be 1.

Q matrix

$x \setminus y$	0	1
0	0.7	0.3
1	0.3	0.7

$1 - 0.7$; $1 - 0.3$

$\times 0.6$, $\times 0.4$

P matrix

$x \setminus y$	0	1
0	0.42	0.18
1	0.12	0.28
	Σ	Σ

$\underline{\mathbf{q}} = [0.54 \quad 0.46]$

2. Consider a DMC whose channel matrix **Q** is given below.
   Suppose the input probability vector is $\underline{\mathbf{p}} = [0.4, 0.3, 0.2, 0.1]$.
   Calculate the missing values in the **Q** matrix.
   Then, find the output probability vector $\underline{\mathbf{q}}$ and the joint pmf matrix **P**.

The missing values can be calculated by using the fact that, in the **Q** matrix, any row sum should be 1.

Q matrix

$x \setminus y$	1	2	3	4
1	0.2	0.2	0.3	0.3
2	0.1	0.4	0.2	0.3
3	0.3	0.1	0.1	0.5
4	0.1	0.1	0.7	0.1

$\times 0.4$, $\times 0.3$, $\times 0.2$, $\times 0.1$

P matrix

$x \setminus y$	1	2	3	4
1	0.08	0.08	0.12	0.12
2	0.03	0.12	0.06	0.09
3	0.06	0.02	0.02	0.10
4	0.01	0.01	0.07	0.01
	Σ	Σ	Σ	Σ

$\underline{\mathbf{q}} = [0.18 \quad 0.23 \quad 0.27 \quad 0.32]$