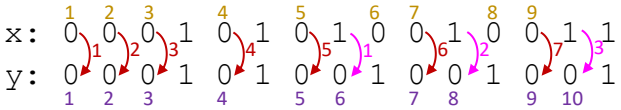
ECS 452: In-Class Exercise # 6 Sol

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups.** Only one submission is needed for each group.
2. **[ENRE] Explanation is not required for the exercise**.
3. **Do not panic.**

Date: 4 / 2 / 2020			
Name	ID (last 3 digits)		
Prapun	5	5	5

1. Consider a DMC whose samples of input and output are provided below



There are 15 samples (pairs of (x,y) values) here.

Estimate the following quantities:

a. $\mathcal{X}$
Channel input alphabet
= support of $X = \{0,1\}$

b. $\mathcal{Y}$
Channel output alphabet
= support of $Y = \{0,1\}$

c. $P[X=0] = \frac{9}{15} = \frac{3}{5} = 0.6$
Among the 15 samples, there are 9 samples with $x = 0$.

d. $p_Y(0) \equiv P[Y=0] = \frac{10}{15} = \frac{2}{3} \approx 0.6667$
Among the 15 samples, there are 10 samples with $y = 0$.

e. $\underline{\mathbf{p}} = [p(0)\ \ p(1)] = \left[\frac{3}{5}\ \frac{2}{5}\right] = [0.6\ 0.4]$

$$p(1) = 1 - p(0) = 1 - \frac{3}{5} = \frac{2}{5}$$

f. $\underline{\mathbf{q}} = [q(0)\ \ q(1)] = \left[\frac{2}{3}\ \frac{1}{3}\right] \approx$ [0.6667 0.3333]

$$q(1) = 1 - q(0) = 1 - \frac{2}{3} = \frac{1}{3}$$

g. $P[Y=0|X=0] = \frac{7}{9} \approx 0.7778$
Among the 9 samples with $x = 0$, there are 7 samples with $y = 0$.

h. $p_{Y|X}(1|0)$
$= P[Y=1|X=0] = 1 - P[Y=0|X=0]$
$= 1 - \frac{7}{9} = \frac{2}{9} \approx 0.2222$

i. $Q(0|1) = P[Y=0|X=1] = \frac{3}{6} = \frac{1}{2} = 0.5$
Among the 15 samples, there are $15 - 9 = 6$ samples with $x = 1$.
Among the 6 samples with $x = 1$, there are 3 samples with $y = 0$.

j. Matrix **Q**
Note that the Q matrix gives the conditional probabilities $P[Y=y|X=x]$. From earlier parts, we have three of such probabilities. The remaining one is

$$P[Y=1|X=1] = 1 - P[Y=0|X=1] = 1 - \frac{1}{2} = \frac{1}{2} = 0.5$$

Therefore,

$$\mathbf{Q} = \begin{matrix} x\backslash y \\ 0 \\ 1 \end{matrix} \begin{matrix} 0 & 1 \end{matrix} \begin{bmatrix} 7/9 & 2/9 \\ 1/2 & 1/2 \end{bmatrix} \approx \begin{matrix} x\backslash y \\ 0 \\ 1 \end{matrix} \begin{matrix} 0 & 1 \end{matrix} \begin{bmatrix} 0.7778 & 0.2222 \\ 0.5 & 0.5 \end{bmatrix}$$

k. $P[X=0, Y=0] = \frac{7}{15} \approx 0.4667$
Among the 15 samples, there are 7 samples with $x = 0$ and $y = 0$.