ECS 452: In-Class Exercise # 4 Sol

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups.** Only one submission is needed for each group.
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: 28 / 1 / 2020			
Name	ID (last 3 digits)		

1. A discrete memoryless source emits three possible messages Yes, No, and OK with probabilities 0.1, 0.1, and 0.8, respectively.

   a. Find the expected codeword length when a binary Huffman code is constructed without extension.

x	$p(x)$	$\ell(x)$
Yes	0.1	2
	0.2	
No	0.1 1	2
OK	0.8	1

$$\mathbb{E}[\ell(X)] = \sum_x \ell(x)p(x)$$
$$= 2 \times 0.1 \times 2 + 1 \times 0.8$$
$$= 0.4 + 0.8$$
$$= 1.2 \text{ [bits per source symbol]}$$

Remark: The problem does not ask us to find the codewords. Only the codeword lengths are needed. Once the tree is formed, we can read the codeword lengths directly.

   b. Find the codeword lengths when binary Huffman code with second-order extension is used to encode this source. Put the values of the corresponding probabilities and the codeword lengths in the table below. (Note that, for brevity, we use Y,N,K to represent Yes, No, and OK, respectively.)

x_1x_2	$p_{X_1,X_2}(x_1,x_2)$	$\ell(x_1,x_2)$
YY	$0.1 \times 0.1 = 0.01$	6
	0.02	
YN	$0.1 \times 0.1 = 0.01$ 0.04	6
	0.12	
YK	$0.1 \times 0.8 = 0.08$	4
	0.20	
	0.36	
NY	$0.1 \times 0.1 = 0.01$ 1	6
	0.02	
NN	$0.1 \times 0.1 = 0.01$	6
NK	$0.1 \times 0.8 = 0.08$	3
KY	$0.8 \times 0.1 = 0.08$	3
	0.16	
KN	$0.8 \times 0.1 = 0.08$	3
KK	$0.8 \times 0.8 = 0.64$	1

Note that there are many possible solutions. This is just one of them. For example, there are four 0.01; any pair is OK for the first grouping.

Remark: The problem does not ask us to find the codewords. Only the codeword lengths are needed. Once the tree is formed, we can read the codeword lengths directly.

   c. Find L_2.
   (This is the expected codeword length **per source symbol** of the Huffman binary code for the second-order extension of this source.)

$$\mathbb{E}[\ell(X_1,X_2)] = \sum_{(x_1,x_2)} \ell(x_1,x_2)p_{X_1,X_2}(x_1,x_2) = 4 \times 6 \times 0.01 + 4 \times 0.08 + 3 \times 3 \times 0.08 + 1 \times 0.64$$
$$= 0.24 + 0.32 + 0.72 + 0.64 = 1.92 \text{ [bits per two source symbols]}$$
$$L_2 = \frac{1.92}{2} = 0.96 \text{ [bits per source symbol]}$$