ECS 452: In-Class Exercise # 2 Sol

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups.** Only one submission is needed for each group.
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: 21 / 1 / 2020			
Name	ID (last 3 digits)		

1. Consider a DMS whose source alphabet is {a,e,ℓ,n,r}.
   The probabilities for these five symbols are shown in the table below:

x	a	e	ℓ	n	r
$p(x)$	0.1	0.2	0.2	0.2	0.3

Consider two codes (for source coding) below.
The left column is for Code A. The right column is for Code B.
The first row defines these codes via their codebooks.

Codebook for Code A

x	a	e	ℓ	n	r
$c(x)$	1	00	010	0110	0111
$\ell(x)$	1	2	3	4	4
$p(x)$	0.1	0.2	0.2	0.2	0.3

Codebook for Code B

x	a	e	ℓ	n	r
$c(x)$	000	100	10	01	11
$\ell(x)$	3	3	2	2	2
$p(x)$	0.1	0.2	0.2	0.2	0.3

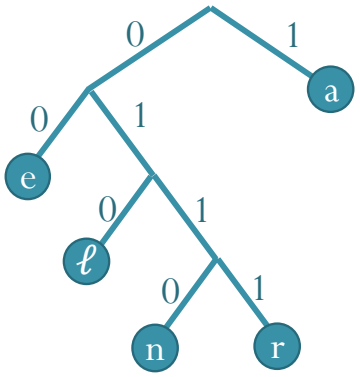
Suppose the DMS above is encoded by Code A.
Find the expected codeword length.

$$\mathbb{E}[\ell(X)] = \sum_x \ell(x)p(x)$$
$$= (1 \times 0.1) + ((2+3+4) \times 0.2) + (4 \times 0.3)$$
$$= 0.1 + 1.8 + 1.2$$
$$= 3.1 \text{ [bits per source symbol]}$$

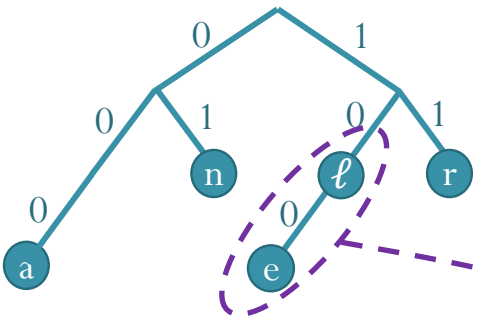
Suppose the DMS above is encoded by Code B.
Find the expected codeword length.

$$\mathbb{E}[\ell(X)] = \sum_x \ell(x)p(x)$$
$$= \big(3 \times (0.1+0.2)\big) + (2 \times (0.2+0.2+0.3))$$
$$= 0.9 + 1.4$$
$$= 2.3 \text{ [bits per source symbol]}$$

Draw the code tree for Code A.



Draw the code tree for Code B.



Is Code A prefix-free?
Method 1: We check all pairs of codewords and see whether, in each pair, a codeword is a prefix of another codeword. Here, yes, the code is prefix-free because no codeword is a prefix of another codeword.
Method 2: We look at the code tree. Code A is a prefix-free code because all of its codewords are leaves (end nodes).

Is Code B prefix-free?
Method 1: We check all pairs of codewords and see whether, in each pair, a codeword is a prefix of another codeword. Here, no, the code is not prefix-free because the codeword 10 is a prefix of the codeword 100.
Method 2: We look at the code tree. Code B is not a prefix-free code because there is an active branch leaving the codeword for ℓ.