Lecture 4 (IT)

Thursday, June 18, 2009
12:53 PM

Boolean Algebra

New system → Value : 0, 1

→ Operations
- Complement
- Addition
- Multiplication

Variable : X, Y, A, B, C

Complement : $\overline{X}, \overline{Y}, \overline{A}$

literal : $X, \overline{X}, Y, \overline{Y}, A, \overline{A}, ..$

Operations

① Complement

$\overline{1} = 0$
$\overline{0} = 1$
} "NOT" operation

② Addition

A	B	A+B
0	0	0
0	1	1
1	0	1
1	1	1

} "OR" operation

③ Multiplication

A	B	A·B
0	0	0
0	1	0
1	0	0
1	1	1

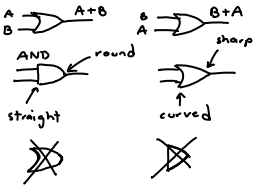
} "AND" operation

Laws

① Commutative :

$A+B = B+A$

$AB = BA$



② Associative :

$A+(B+C) = (A+B)+C = A+B+C$

$A(BC) = (AB)C$

③ Distributive :

$A(B+C) = AB+AC$

↑ A, not B

$A+(BC) = (A+B)\cdot(A+C)$ *

Rules

① $A+0 = A$

$A \cdot 1 = A$

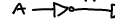
② $A+1 = 1$ *

$A \cdot 0 = 0$ ←

③ $A+A = A$ *

$A \cdot A = A$

④ $\overline{\overline{A}} = A$

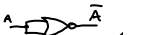


Ex A NAND

$\overline{A \cdot A} =$

Ex A NOR A

$\overline{A+A} =$



Ex A + AB =

=

$\overline{\overline{A}} = A$

$A = \overline{A}$

$\overline{A} = \overline{A}$

$A = \overline{A}$

$\overline{A} = \overline{A}$

same

$A \cdot 1 + A \cdot B = A \cdot (1 + B)$

$A \cdot 1 = A$

Ex $A \cdot (A + B)$

$= A \cdot A + A \cdot B$

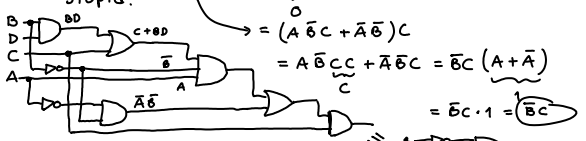
$= A + A \cdot B = A$

Ex. Simplify

$(A\bar{B}(C + BD) + \bar{A}\bar{B})C = (A\bar{B}C + A\bar{B}BD) + \bar{A}\bar{B})C$

($A\bar{B}BD = 0$, $0 + 0$)

Stupid!



$= (A\bar{B}C + \bar{A}\bar{B})C$

$= A\bar{B}CC + \bar{A}\bar{B}C = \bar{B}C(A + \bar{A})$

($CC = C$)

$= \bar{B}C \cdot 1 = \bar{B}C$

$\equiv$

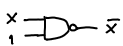
Ex. Simplify

$\bar{A}B + \bar{A}B\bar{C} + \bar{A}BCD + \bar{A}B\bar{C}\bar{D}E$

$= \bar{A}B(1 + \bar{C} + CD + \bar{C}\bar{D}E)$

$= \bar{A}B \cdot 1 = \bar{A}B$

Ex. X NAND 1 = ?



$\overline{X \cdot 1} = \bar{X}$

X NOR 0 = ?

$\overline{X + 0} = \bar{X}$

X NAND X = $\bar{X}$

X NOR X = $\bar{X}$

$\bar{X}$ = X NAND 1

= X NOR 0

= X NAND X

= X NOR X

DeMorgan's Theorem

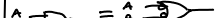
① complement of a product = sum of the complement of individ. variable

$\overline{X \cdot Y \cdot Z} = \bar{X} + \bar{Y} + \bar{Z}$

$\overline{X\bar{Y}Z} = \bar{X} + \overline{\bar{Y}} + \bar{Z}$

$= \bar{X} + Y + \bar{Z}$

$\overline{A \cdot B} = \bar{A} + \bar{B}$



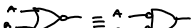
NAND Negative-OR

② complement of a sum
= product of the complements of individual variables

$\overline{X+Y+Z} = \bar{X} \cdot \bar{Y} \cdot \bar{Z}$

$\overline{A+B} = \bar{A} \cdot \bar{B}$

Next time: K-map



NOR Negative-AND