Lecture 4 - CS
Thursday, June 18, 2009
8:59 AM

Boolean Algebra

* Different from elementary (high school) algebra.

* New System
  - value : 0, 1 (~~T, F~~) (~~F, T~~)
  - Operation
    - addition
    - multiplication
    - complement *

variable: A, B, X, Y

Complement : $\bar{A}, \bar{B}, \bar{X}, \bar{Y}$

$\bar{A}$ is the complement of A.

Literal : A, $\bar{A}$, B, $\bar{B}$

variable or its complement

Operation

1. Complement

   If A = 1, then $\bar{A}$ = 0.

   If A = 0, then $\bar{A}$ = 1.

2. Multiplication

A	B	AB
0	0	0
0	1	0
1	0	0
1	1	1

⇒ Same as "AND"

3. Addition

A	B	A+B
0	0	0
0	1	1
1	0	1
1	1	1

⇒ Same as "OR"

Laws :

Commutative law

A+B = B+A

AB = BA

Proof

A	B	A+B	B+A
0	0	0	0
0	1	1	1
1	0	1	1
1	1	1	1

*

Associative Laws

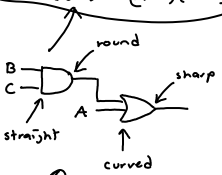
$A+(B+C) = (A+B)+C$

$A(BC) = (AB)C$

Distributive Laws

$A(B+C) = AB+AC$

$A+(BC) = (A+B)(A+C)$ *



Rules

1. A + 0 =

   A · 1 =

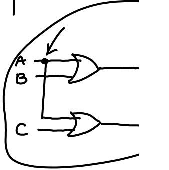
2. A + 1 =

   A · 0 =

3. A + A =

   A · A =

≡



A

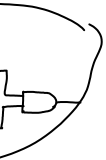
$\cdot A$ }

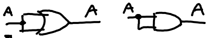
1 *

0

A *

A *



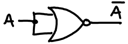


(4) $\overline{\overline{A}} = A$

Ex.

$\overline{A \cdot A} = \overline{A}$

↑ Idempotency



$\overline{A + A} = \overline{A}$

(5) Complements

$A + \overline{A} = 1$ *

$A \cdot \overline{A} = 0$ *

Ex.

$$A + AB = A \cdot 1 + A \cdot B = A \cdot (1 + B) = A \cdot 1 = A$$

Ex.

$$A \cdot (A + B) = A \cdot A + A \cdot B = A + AB = A$$

$A + (AB) = A$

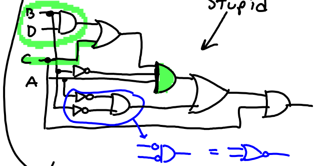
$A \cdot (A + B) = A$

Dualities

Can I get new identities by $+ \to \cdot$, $\cdot \to +$ | $1 \to 0$, $0 \to 1$? } Yes!

Ex.
Simplify

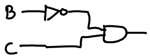
$$(A\overline{B}(C + BD) + \overline{A}\overline{B})C$$



$$((A\overline{B}C + A\overline{B}\underbrace{BD}_{0}) + \overline{A}\overline{B})C$$

$$= (A\overline{B}C + \overline{A}\overline{B})C = A\overline{B}C + \overline{A}\overline{B}C$$

$$= \overline{B}C(A + \overline{A}) = \overline{B}C \cdot 1 = \overline{B}C$$



Ex

$$\overline{A}B + \overline{A}B\overline{C} + \overline{A}BCD$$

$$= \overline{A}B(\underbrace{1 + \overline{C} + CD}_{1})$$

$$= \overline{A}B$$

Next time: K-map!