Sirindhorn International Institute of Technology
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ECS 371: Boolean Algebra and Digital Logic (1)

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Disclaimer: The note should be used as a supplementary material and should not be considered as a replacement for the main textbook.

1 Boolean Operations and Expressions

1.1. Boolean algebra (or Boolean logic) is the algebra of two values. These are usually taken to be 0 and 1.

- F and T, false and true, etc. are also in common use.
- It was developed in 1854 by George Boole in his book *An Investigation of the Laws of Thought*.
- It provides a concise way to express the operation of a logic circuit formed by a combination of logic gates.

1.2. Variable:

(a) A **variable** is a symbol (usually an italic uppercase letter or word) used to represent an action, a condition, or data.

- Any single variable can have only a 1 or a 0 value.

(b) The **complement** is the inverse of a variable and is indicated by a bar over the variable (overbar).

- For example, the complement of the variable A is $\overline{A}$.
- The complement of the variable A is read as "not A" or "A bar".
- Sometimes a prime symbol rather than an overbar is used to denote the complement of a variable; for example, B' indicates the complement of B.
- Other inversion symbols: $\neg X, /X, \sim X$

(c) A **literal** is a variable or the complement of a variable.

1.3. Boolean Operations: Logical operations are equivalent to the basic gate operations that you are familiar with.

(a) Boolean addition is equivalent to the OR operation.

(b) Boolean multiplication is equivalent to the AND operation.

(c) Complementation.

- Just like in normal arithmetic, multiplication (AND in boolean logic) has precedence over addition (OR in boolean logic).
  - $W \cdot X + Y \cdot Z$ means $(W \cdot X) + (Y \cdot Z)$
  - Use parentheses to avoid ambiguity.

1.4. Sum and Product Terms:

(a) A **sum term** is a sum of literals.

- A sum term is equal to 1 if and only if one or more of the literals in the term are 1.
- A sum term is equal to 0 if and only if each of the literals is 0.

(b) A **product term** is the product of literals.

- A product term is equal to 1 if and only if each of the literals in the term is 1.
- A product term is equal to 0 if and only if one or more of the literals are 0.

1.5. The Basic Laws of Boolean Algebra:

(a) Commutative laws:

(i) Commutative law of addition:

$$A + B = B + A.$$

(ii) Commutative law of multiplication:

$$AB = BA.$$

(b) Associative Laws:

(i) Associative law of addition:

$$A + (B + C) = (A + B) + C.$$

(ii) Associative law of multiplication:

$$A(BC) = (AB)C.$$

(c) Distributive Law:

(i) $A(B + C) = AB + BC$

(ii) $A + BC = (A + B)(A + C)$

- The distributive laws also express the process of factoring.

1.6. Rules of Boolean Algebra:

(a) Identities:

(i) $A + 0 = A$

(ii) $A \cdot 1 = A$

(b) Null Elements:

(i) $A + 1 = 1$

(ii) $A \cdot 0 = 0$

(c) Idempotency:

(i) $A + A = A$

(ii) $A \cdot A = A$

(d) Involution: $\overline{\overline{A}} = A$

(e) Complements

(i) $A + \overline{A} = 1$

(ii) $A \cdot \overline{A} = 0$

(f) Covering:

(i) $A + AB = A$

(ii) $A \cdot (A + B) = A$

1.7. DeMorgan's theorem

(a) The complement of a product of variables is equal to the sum of the complements of the variables.

- The complement of two or more ANDed variables is equivalent to the OR of the complements of the individual variables.
- Examples:
  - "NAND ≡ Negative-OR":

$$\overline{AB} = \overline{A} + \overline{B}$$

  - $\overline{ABC} = \overline{A} + \overline{B} + \overline{C}$

(b) The complement of a sum of variables is equal to the product of the complements of the variables.

- The complement of two or more ORed variables is equivalent to the AND of the complements of the individual variables.
- Examples:
  - "NOR ≡ Negative-AND":

$$\overline{A + B} = \overline{A}\,\overline{B}$$

  - $\overline{A + B + C} = \overline{A}\,\overline{B}\,\overline{C}$