ECS 332: In-Class Exercise # 7 - Sol

Date: 1 3 / 0 9 / 2019			
Name	ID (last 3 digits)		

Instructions

1. Separate into groups of no more than three students each.
   The group cannot be the same as any of your former groups.
2. Explanation is not required for this exercise [ENRE]
3. **Do not panic.**

1. Consider an LTI communication channel.
   Suppose when we put
   $$x(t) = 4\cos(\pi t) + 2\cos(2\pi t) + \cos(4\pi t) + 0.5\cos(6\pi t) + \cos(8\pi t) + 1$$
   into this channel, we get
   $$y(t) = 0.5\cos(\pi t) + e^{j2\pi t} + 2\sin(4\pi t) + 4\cos(6\pi t) + 8$$
   as its output.

   Let $H(f)$ be the frequency response of the channel that satisfies the above input-output relation.

   Find the value of $H(f)$ at each of the frequencies in the table below.

f	-3	-1	1	2	3
$H(f)$	8	0	1	$-2j$	8

In class, we have seen two properties of LTI system:

- Suppose we know that
  $$\boxed{ae^{j2\pi f_0 t} \to \boxed{H(f)} \to be^{j2\pi f_0 t}.}$$
  Then, we can infer that
  $$\boxed{H(f_0) = \frac{b}{a}.}$$
- Suppose we know that
  $$\boxed{a\cos(2\pi f_0 t) \to \boxed{H(f)} \to b\cos(2\pi f_0 t + \theta).}$$
  Then, we can infer that
  $$\boxed{H(f_0) = \frac{b}{a}e^{j\theta} \text{ and } H(-f_0) = \frac{b}{a}e^{-j\theta}.}$$

Here, we are given f_0. Hence, the key is to find which parts of the input $x(t)$ and output $y(t)$ correspond to the given frequency.

- For $f_0 = \pm 3$, we are given that
  $$0.5\cos(6\pi t) \to H(f) \to 4\cos(6\pi t + 0).$$
  Therefore, $H(3) = \frac{4}{0.5}e^{j0} = 8$ and $H(-3) = \frac{4}{0.5}e^{-j0} = 8$.
- For $f_0 = \pm 1$, we are given that
  $$2\cos(2\pi t) \to H(f) \to e^{j2\pi t}.$$
  This does not directly fit the known forms above. However, once we apply the Euler's formula, we get
  $$2\cos(2\pi t) = e^{j2\pi t} + e^{-j2\pi t}.$$

  Therefore, we are given that
  $$e^{j2\pi t} + e^{-j2\pi t} \to H(f) \to e^{j2\pi t} + 0e^{-j2\pi t}.$$

  Therefore, $H(1) = \frac{1}{1} = 1$ and $H(-1) = \frac{0}{1} = 0$.
- For $f_0 = \pm 2$, we are given that
  $$\cos(4\pi t) \to H(f) \to 2\sin(4\pi t).$$
  Recall that $\sin(x) = \cos(x - 90°)$. Therefore, we know that
  $$\cos(4\pi t) \to H(f) \to 2\cos(4\pi t - 90°).$$
  Therefore, $H(3) = \frac{2}{1}e^{j(-90°)} = 2(-j) = -2j$.