ECS 332: In-Class Exercise # 6 - Sol

Instructions

1. Separate into groups of no more than three students each.
   The group cannot be the same as any of your former groups.
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: 11 / 09 / 2019

Name	ID (last 3 digits)		

In this problem, we have three "devices".

- $\boxed{(\cdot)^2}$ is a "square" device. As the name suggests, its output is created by squaring its input in the **time** domain.
- $\boxed{H_1(f)}$ is an LTI device whose **frequency response** is $H_1(f) = \begin{cases} 1, & |f| < 234, \\ 0, & \text{otherwise.} \end{cases}$
- $\boxed{H_2(f)}$ is an LTI device whose **frequency response** is $H_2(f) = \begin{cases} 1, & |f| > 234, \\ 0, & \text{otherwise.} \end{cases}$

✶ Recall that

$$e^{j2\pi f_0 t} \to \boxed{H(f)} \to H(f_0)e^{j2\pi f_0 t}$$

Find the output $y(t)$ for each of the systems below.

$j2\pi f_0 t = 332\pi t \Rightarrow f_0 = 166$

(a) $x(t) = e^{332\pi t} \longrightarrow \boxed{H_1(f)} \longrightarrow y(t)$

$H_1(166) = 1$ because $|166| < 234$.

$$y(t) = H_1(f_0)e^{j2\pi f_0 t} = H_1(166)e^{j2\pi(166)t} = 1e^{j332\pi t} = 1e^{j332\pi t}$$

✧ Recall that

$$\cos(2\pi f_0 t) \to \boxed{H(f)} \to \frac{1}{2}H(f_0)e^{j2\pi f_0 t} + \frac{1}{2}H(-f_0)e^{-j2\pi f_0 t}$$

$$= H(f_0)\cos(2\pi f_0 t)$$

when $H(f)$ is an even function which is the case here

(b) $x(t) = \cos(332\pi t) \longrightarrow \boxed{H_1(f)} \longrightarrow y(t)$

$$y(t) = H_1(f_0)\cos(2\pi f_0 t) = \overbrace{H_1(166)}^{1}\cos(2\pi(166)t) = \cos(332\pi t)$$

(c) $x(t) = \cos(332\pi t) \longrightarrow \boxed{H_2(f)} \longrightarrow y(t)$

$$y(t) = H_2(f_0)\cos(2\pi f_0 t) = \overbrace{H_2(166)}^{0}\cos(2\pi(166)t) = 0$$

(d) $x(t) = \cos(332\pi t) \longrightarrow \boxed{(\cdot)^2} \xrightarrow{x^2(t)} \boxed{H_1(f)} \longrightarrow y(t)$

$$x^2(t) = \cos^2(332\pi t) = \left(\frac{e^{j332\pi t} + e^{-j332\pi t}}{2}\right)^2 = \frac{1}{4}e^{j2\pi(332)t} + \frac{1}{2} + \frac{1}{4}e^{j2\pi(-332)t}$$

One can view the constant $\frac{1}{2}$ as a complex-expo. function $\frac{1}{2}e^{j2\pi(0)t}$ whose freq. is 0

So, $x^2(t)$ is simply a linear combination of complex-exponential functions. Therefore, we can apply our ✶ to each term.

$$y(t) = \frac{1}{4}\underbrace{H_1(332)}_{0}e^{j2\pi(332)t} + \frac{1}{2}\overbrace{H_1(0)}^{1} + \frac{1}{4}\underbrace{H_1(-332)}_{0}e^{j2\pi(-332)t} = \frac{1}{2}$$

(e) $x(t) = \cos(332\pi t) \longrightarrow \boxed{(\cdot)^2} \xrightarrow{x^2(t)} \boxed{H_2(f)} \longrightarrow y(t)$

Here, we can still use the expression of $x^2(t)$ derived in the previous part. However, we have to change the frequency response of the device from $H_1(f)$ to $H_2(f)$.

$$y(t) = \frac{1}{4}\underbrace{H_2(332)}_{1}e^{j2\pi(332)t} + \frac{1}{2}\underbrace{H_2(0)}_{0} + \frac{1}{4}\underbrace{H_2(-332)}_{1}e^{j2\pi(-332)t} = \frac{1}{4}e^{j2\pi(332)t} + \frac{1}{4}e^{j2\pi(-332)t} = \frac{1}{2}\cos(664\pi t)$$