ECS 332: In-Class Exercise # 1 Sol

Date: 21 / 08 / 2019			
Name	ID (last 3 digits)		
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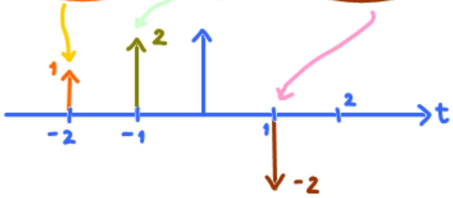
Instructions

1. Separate into groups of no more than three students each.
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

1) Plot $\delta(t+2)+2\delta(t+1)-2\delta(t-1)$.



This is the plot of $A\delta(t-T)$:

2) Evaluate the following integrals:

a) $\int_1^7 \delta(t)dt = \int_1^7 1\,\delta(t)dt = 0$

$\left.\begin{matrix} A=[1,7] \\ c=0 \end{matrix}\right\} \Rightarrow c \notin A$

$g(t) \equiv 1$

b) $\int_1^7 \delta(t-3)dt = \int_1^7 1\,\delta(t-3)dt = g(3) = 1$

$\left.\begin{matrix} A=[1,7] \\ c=3 \end{matrix}\right\} \Rightarrow c \in A$

$g(t) \equiv 1$

Extended sifting property:

$$\int_A g(t)\delta(t-c)dt = \begin{cases} g(c), & c\in A, \\ 0, & c\notin A. \end{cases}$$

c) $\int_1^7 \cos\left(\frac{\pi}{2}t\right)\delta(t)dt = 0$

$\left.\begin{matrix} A=[1,7] \\ c=0 \end{matrix}\right\} \Rightarrow c \notin A$

$g(t) = \cos\left(\frac{\pi}{2}t\right)$

d) $\int_1^7 \cos\left(\frac{\pi}{2}t\right)\delta(t-3)dt = g(3) = \cos\left(\frac{3\pi}{2}\right) = 0$

$\left.\begin{matrix} A=[1,7] \\ c=3 \end{matrix}\right\} \Rightarrow c \in A$

$g(t) = \cos\left(\frac{\pi}{2}t\right)$

e) $\int_0^\infty \delta(t+2)+2\delta(t+1)-2\delta(t-1)dt = \int_0^\infty \delta(t+2)dt + 2\int_0^\infty \delta(t+1)dt - 2\int_0^\infty \delta(t-1)dt$

$A=[0,\infty)$

$-2 \notin A$, $-1 \notin A$, $1 \in A$

$= 0 + 2(0) - 2(1)$

$= -2$

f) (optional) $\int_{-\infty}^{\infty} \delta(t^2-2t)dt$

Come discuss with Dr. Prapun if you think you have found a way to solve this.

A good start is to find the roots of t^2-2t

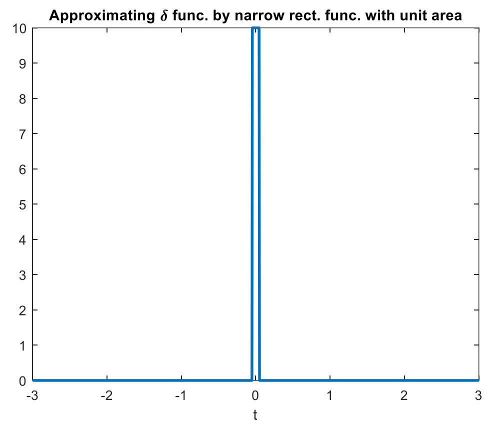
$\hookrightarrow t=0,2.$

However, this is not enough. The answer will also depend on the slope of t^2-2t at $t=0,2$ as well.

In the optional question, we would like to calculate $\int_{-\infty}^{\infty} \delta\left(g(t)\right)dt$ where $g(t) = t^2 - 2t$.

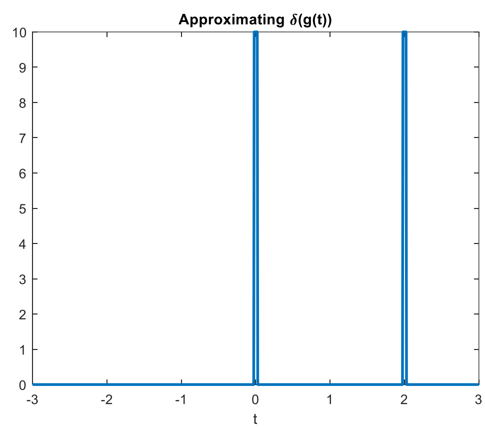
Here are some hints.

Let's look at the limiting approximation of $\delta(t)$. Consider a rectangular function centered at origin whose width is ε. To be a delta function, we need the area = 1; therefore, the height must be $\frac{1}{\varepsilon}$. In the MATLAB plot below, we use $\varepsilon = 0.1$. As expected, there is a "spike" at $t = 0$.




```
close all; clear all;
ep = 1e-1;
t = linspace(-3,3,1e3);

% The rectangular func. that approximates the delta func.
d = 1/ep * rectangularPulse(-ep/2,ep/2,t);
plot(t,d,'LineWidth',2)
title('Approximating \delta func. by narrow rect. func. with unit area')
xlabel('t')

g = t.*(t-2);

figure
plot(t,g,'LineWidth',2)
ylim([-3,3])
grid on
title('g(t)')
xlabel('t')

figure
% plugging g(t) into the delta function
deltag = 1/ep * rectangularPulse(-ep/2,ep/2,g);
plot(t,deltag,'LineWidth',2)
title('Approximating \delta(g(t))')
xlabel('t')
```

Now, let's try to plot $\delta\left(g(t)\right)$ where $g(t) = t^2 - 2t$.



Note that there are two "spikes" at $t = 0$ and $t = 2$. This is expected because we know that the spikes will show up when the argument of $\delta(\cdot)$ is 0. Here, $g(t) = t^2 - 2t$ is zero at $t = 0$ and $t = 2$.

The area under the graph above approximates $\int_{-\infty}^{\infty} \delta\left(g(t)\right)dt$. This area is the sum of the areas under the two spikes. Note, however, that the area under each spike is not one anymore; the spikes seem to be narrower. How can we find their areas? (Furthermore, can we eliminate MATLAB from this calculation?)