ECS 332: In-Class Exercise # 1 **Sol**

Date: **21** / **08** / 2019

Name		ID (last 3 digits)		
Prapun		**5**	**5**	**5**

Instructions

1. Separate into groups of no more than three students each.
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
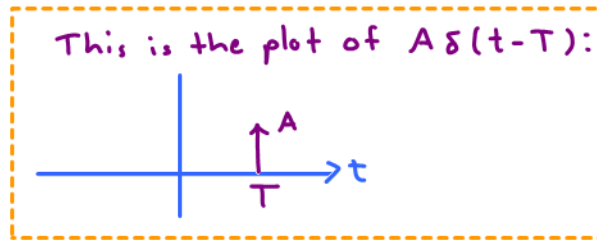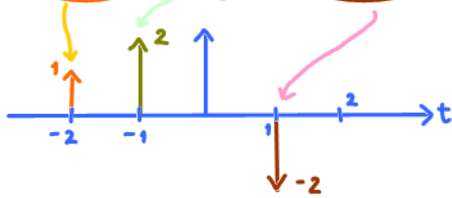3. **Do not panic.**

1) Plot $\delta(t+2)+2\delta(t+1)-2\delta(t-1)$.

This is the plot of $A\delta(t-T)$:

2) Evaluate the following integrals:

a) $\int_{1}^{7}\delta(t)dt = \int_{1}^{7} 1 \cdot \delta(t)dt = 0$

$\left.\begin{array}{l} A = [1,7] \\ c = 0 \end{array}\right\} \Rightarrow c \notin A$

$g(t) \equiv 1$

b) $\int_{1}^{7}\delta(t-3)dt = \int_{1}^{7} 1 \cdot \delta(t-3)dt = g(3) = 1$

$\left.\begin{array}{l} A = [1,7] \\ c = 3 \end{array}\right\} \Rightarrow c \in A$

$g(t) \equiv 1$

Extended sifting property:

$$\int_{A} g(t)\,\delta(t-c)\,dt = \begin{cases} g(c), & c \in A, \\ 0, & c \notin A. \end{cases}$$

c) $\int_{1}^{7}\cos\left(\frac{\pi}{2}t\right)\delta(t)dt = 0$

$\left.\begin{array}{l} A = [1,7] \\ c = 0 \end{array}\right\} \Rightarrow c \notin A$

$g(t) = \cos\left(\frac{\pi}{2}t\right)$

d) $\int_{1}^{7}\cos\left(\frac{\pi}{2}t\right)\delta(t-3)dt = g(3) = \cos\left(\frac{3\pi}{2}\right) = 0$

$\left.\begin{array}{l} A = [1,7] \\ c = 3 \end{array}\right\} \Rightarrow c \in A$

$g(t) = \cos\left(\frac{\pi}{2}t\right)$

e) $\int_{0}^{\infty}\delta(t+2)+2\delta(t+1)-2\delta(t-1)dt = \int_{0}^{\infty}\delta(t+2)dt + 2\int_{0}^{\infty}\delta(t+1)dt - 2\int_{0}^{\infty}\delta(t-1)dt$

$A = [0,\infty)$

$\underset{-2\notin A}{= 0} \quad \underset{\substack{-1\notin A \\ +2(0)}}{} \quad \underset{\substack{1\in A \\ -2(1)}}{}$

$= -2$

f) (optional) $\int_{-\infty}^{\infty}\delta(t^2-2t)dt$

Come discuss with Dr. Prapun if you think you have found a way to solve this.

A good start is to find the roots of t^2-2t
  ↳ $t = 0, 2$.

However, this is not enough. The answer will also depend on the slope of t^2-2t at $t=0,2$ as well.

In the optional question, we would like to calculate $\int_{-\infty}^{\infty} \delta(g(t)) dt$ where $g(t) = t^2 - 2t$.

Here are some hints.

Let's look at the limiting approximation of $\delta(t)$. Consider a rectangular function centered at origin whose width is ε. To be a delta function, we need the area = 1; therefore, the height must be $\dfrac{1}{\varepsilon}$. In the MATLAB plot below, we use $\varepsilon = 0.1$. As expected, there is a "spike" at $t = 0$.

Approximating δ func. by narrow rect. func. with unit area
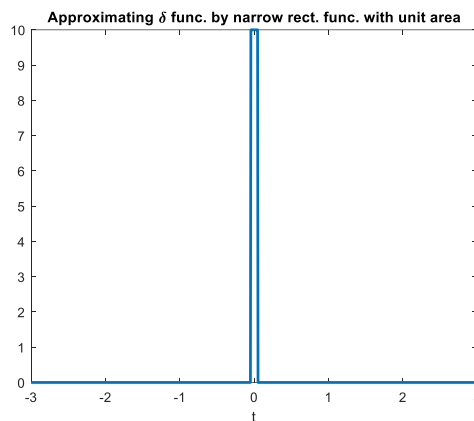
```
close all; clear all;
ep = 1e-1;
t = linspace(-3,3,1e3);

% The rectangular func. that approximates the delta func.
d = 1/ep * rectangularPulse(-ep/2,ep/2,t);
plot(t,d,'LineWidth',2)
title('Approximating \delta func. by narrow rect. func. with unit area')
xlabel('t')

g = t.*(t-2);

figure
plot(t,g,'LineWidth',2)
ylim([-3,3])
grid on
title('g(t)')
xlabel('t')

figure
% plugging g(t) into the delta function
deltag = 1/ep * rectangularPulse(-ep/2,ep/2,g);
plot(t,deltag,'LineWidth',2)
title('Approximating \delta(g(t))')
xlabel('t')
```
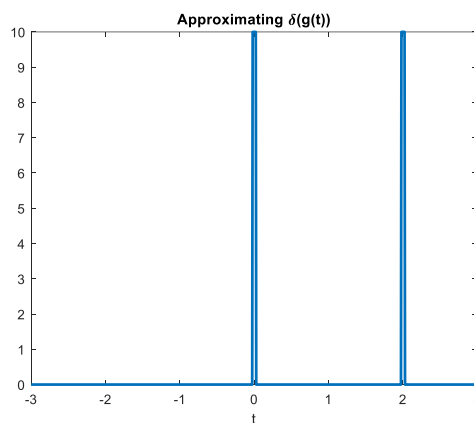
Now, let's try to plot $\delta(g(t))$ where $g(t) = t^2 - 2t$.

Approximating δ(g(t))

Note that there are two "spikes" at $t = 0$ and $t = 2$. This is expected because we know that the spikes will show up when the argument of $\delta(\cdot)$ is 0. Here, $g(t) = t^2 - 2t$ is zero at $t = 0$ and $t = 2$.

The area under the graph above approximates $\int_{-\infty}^{\infty} \delta(g(t)) dt$. This area is the sum of the areas under the two spikes. Note, however, that the area under each spike is not one anymore; the spikes seem to be narrower. How can we find their areas? (Furthermore, can we eliminate MATLAB from this calculation?)

ECS 332: In-Class Exercise # 2 - Sol

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as your group** for the previous exercise.
2. [ENRE] = Explanation is not required for this exercise.
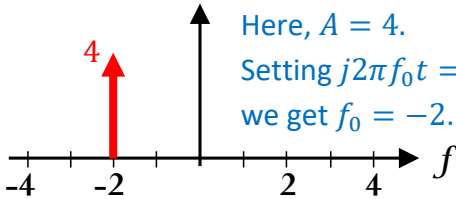3. **Do not panic.**

1. [ENRPr] Consider each $g(t)$ defined below.

Let $G(f)$ be its Fourier transform. Plot $G(f)$ from $f = -4$ to $f = 4$ Hz.

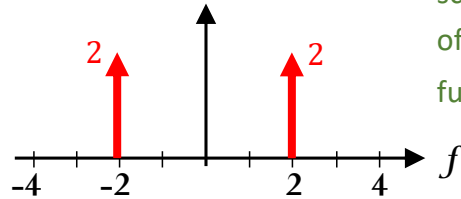a. $g(t) = 4e^{-j4\pi t}$

$$Ae^{j2\pi f_0 t} \overset{\mathcal{F}}{\rightleftharpoons} A\delta(f - f_0)$$

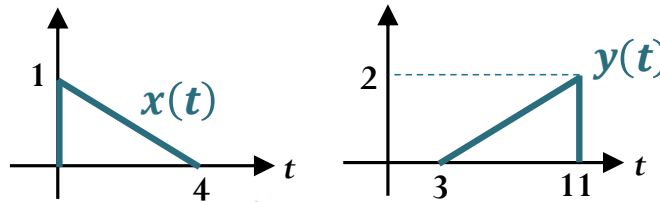Here, $A = 4$.
Setting $j2\pi f_0 t = -j4\pi t$,
we get $f_0 = -2$.

b. $g(t) = 4\cos(4\pi t)$

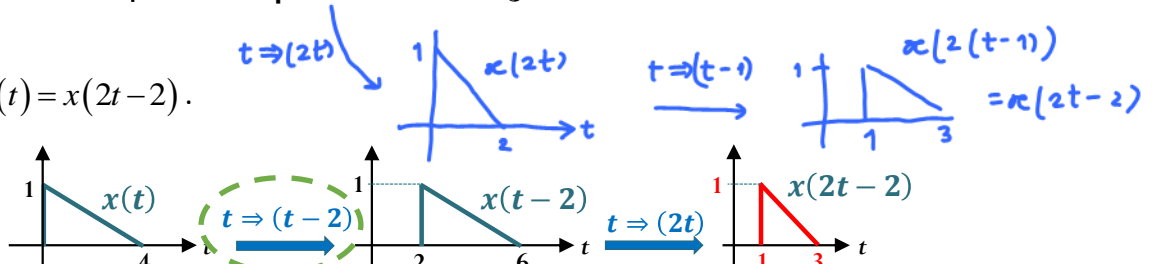Don't forget to scale the size of each δ-function by $\frac{1}{2}$.

$$A\cos(2\pi f_0 t) \overset{\mathcal{F}}{\rightleftharpoons} \frac{A}{2}\delta(f - (-f_0)) + \frac{A}{2}\delta(f - f_0)$$

2. [ENRPr] Signals $x(t)$ and $y(t)$ are plotted below.

a) Plot the signal $w(t) = x(2t - 2)$.
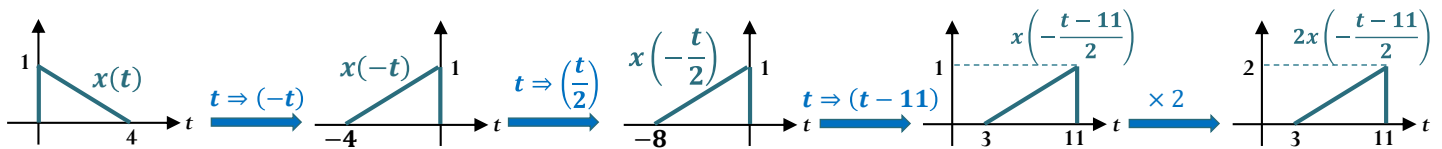
$t \Rightarrow (2t)$

$x(2t)$

$t \Rightarrow (t-1)$

$x(2(t-1))$
$= x(2t-2)$

This notation means we replace t by $t - 2$.

b) Suppose $y(t) = c_1 x(c_2 t + c_3)$. Find the values of the constants $c_1, c_2,$ and c_3.

$$c_1 = \underline{\ 2\ }, c_2 = \underline{\ -0.5\ }, c_3 = \underline{\ 5.5\ }.$$
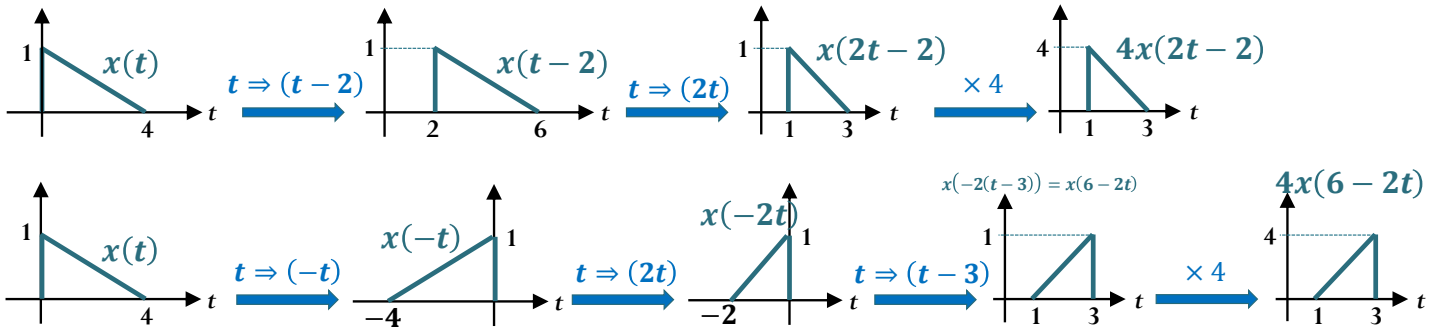

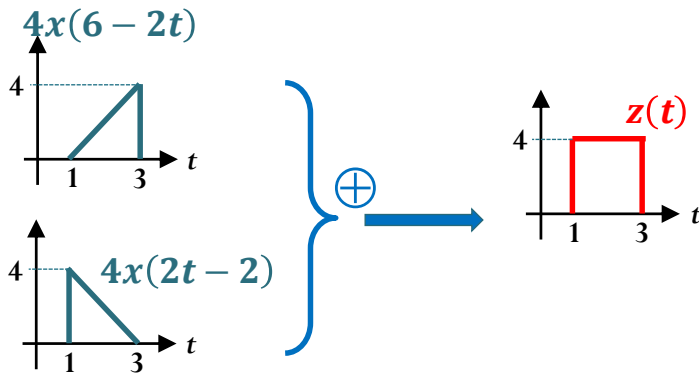$$y(t) = 2x\left(-\frac{t - 11}{2}\right) = 2x(-0.5t + 5.5).$$

Caution: One common mistake is that, in the third step, when we shift the graph to the right by 11 units, we can't just put "-11" blindly into the expression and get $x\left(-\frac{t}{2} - 11\right)$; we need to replace t by $t - 11$.

c) Suppose $z(t) = 4x(2t-2) + 4x(6-2t)$. Plot $z(t)$.
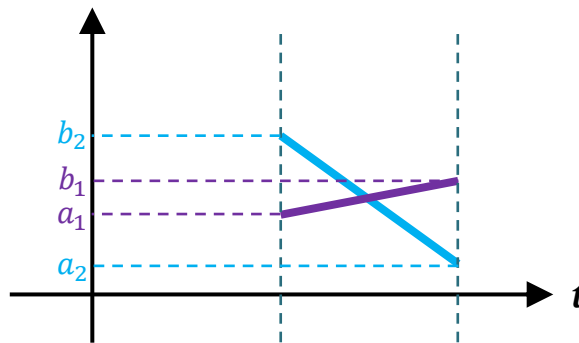
First, we plot $4x(2t-2)$ and $4x(6-2t)$.

Next, we combine (add) the two plots.

Remark: Adding two straight lines.

Given the graphs of two straight lines over an interval, it is easy to find their sum.

For example, suppose we want to find the sum of the two straight lines below:

First, note that the sum is still a straight line.

To see this, let's assume that the two original straight lines are given by $m_1 t + c_1$ and $m_2 t + c_2$. Then, their sum is $(m_1 + m_2)t + (c_1 + c_2)$ which is still a straight line.

To draw a straight line, it is enough to find two points that it passes. In the example above, this can be done easily at the two boundaries of the interval.
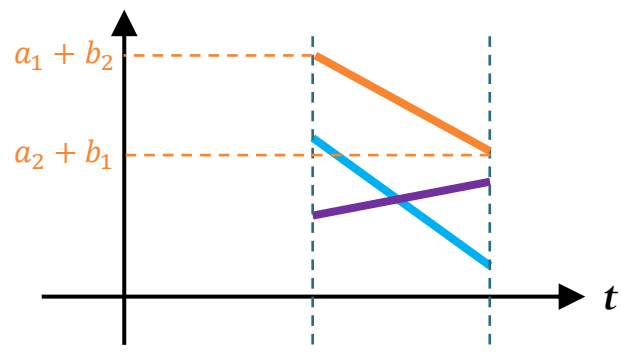
ECS 332: In-Class Exercise # 3 - Sol

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups.**
2. **[ENRE]** = Explanation is not required for this exercise.
3. **Do not panic.**

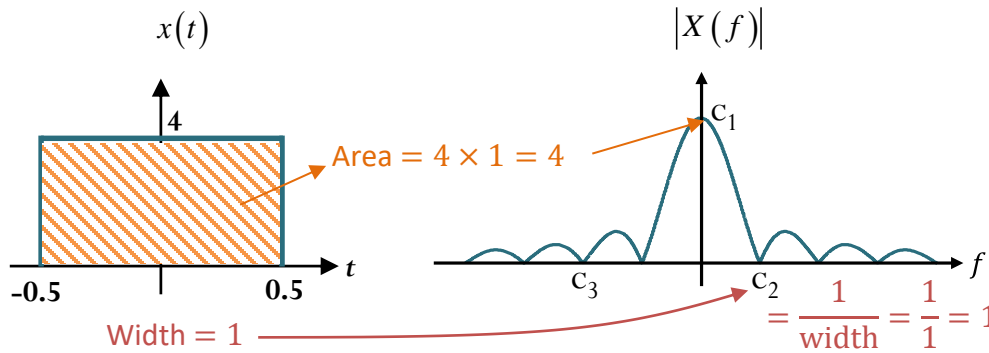| Name		ID (last 3 digits)		

1. A signal and its magnitude spectrum are plotted below.

$x(t)$

4

Area $= 4 \times 1 = 4$

-0.5 0.5 t

Width $= 1$

$|X(f)|$

C_1

C_3 C_2 $\dfrac{1}{\text{width}} = \dfrac{1}{1} = 1$ f

For this question, we apply observation 2.13 in the lecture notes:

(0) The Fourier transform of a rectangular function is a sinc function.

(i) The height of the sinc function's peak is the same as the area under the rectangular function.

(ii) The first zero crossing of the sinc function occurs at 1/(width of the rectangular function).
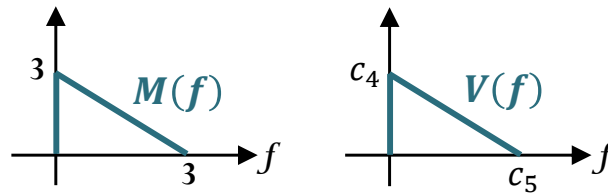
Note also that in the magnitude plot, the negative-valued parts of the sinc function are flipped up.

Find the values of the constants (corresponding to some zeroes and the peak value) shown in the plots.

$$c_1 = \underline{\quad 4 \quad}, c_2 = \underline{\quad 1 \quad}, c_3 = \underline{\quad -2 \quad}.$$

2. Consider a signal $m(t)$ and another signal $v(t) = m(2t)$.

Their corresponding Fourier transforms are plotted below.

3 $M(f)$ 3 f

c_4 $V(f)$ c_5 f

Find the values of the constants in the plot of $V(f)$:

$$c_4 = \underline{\quad \dfrac{3}{2} \quad}, c_5 = \underline{\quad 6 \quad}$$

For $v(t) = m(2t)$, by the scale-change theorem, we have

$$V(f) = \frac{1}{|2|} M\left(\frac{f}{2}\right) = \frac{1}{2} M\left(\frac{f}{2}\right).$$

In the previous exercise, we worked on time manipulation. Note that, back then, "time" was just a dummy variable. Here, it's the frequency f. We can get $M\left(\frac{f}{2}\right)$ from $M(f)$ by replacing f by $\frac{f}{2}$; graphically, this is a horizontal expansion by a factor of 2. This implies $c_5 = 2 \times 3 = 6$.

Finally, the $\frac{1}{2}$ in the front simply scales the height of graph by a factor of $\frac{1}{2}$. This implies $c_4 = \frac{1}{2} \times 3 = \frac{3}{2}$.

ECS 332: In-Class Exercise # 4 - Sol

Instructions

1. Separate into groups of no more than three students each.
   The group cannot be the same as any of your former groups.
2. **[ENRE]** = Explanation is not required for this exercise
3. **Do not panic.**

Date: <u>0</u> <u>4</u> / <u>0</u> <u>9</u> / 2019

Name		ID (last 3 digits)		

[ENRE] A signal and its magnitude spectrum are plotted below.

$x(t)$ | $|X(f)|$

3

Area $= 3 \times 4 = 12$

C_1

-0.5   3.5 t

Width = 4

$C_2 = \dfrac{1}{\text{width}} = \dfrac{1}{4}$ 1   1 f
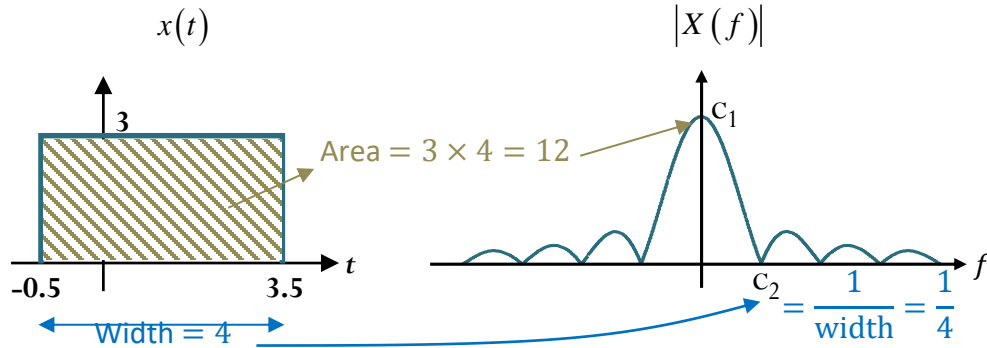
1. Find the values of the constants (corresponding to some zero and the peak value) shown in the plots.

$$c_1 = \underline{\quad 12 \quad}, c_2 = \underline{\quad \frac{1}{4} \quad}.$$

This problem is similar to the one we have worked on in the previous exercise. However, the rectangular function is not centered at $t = 0$; it is time-shifted. From the time-shift property (2.31), we know that the magnitude spectrum plot won't be affected by this time-shifting. So, we can still use 2.13. In particular,

(0) The Fourier transform of a rectangular function is a sinc function.
(i) The height of the sinc function's peak is the same as the area under the rectangular function.
(ii) The first zero crossing of the sinc function occurs at 1/(width of the rectangular function).

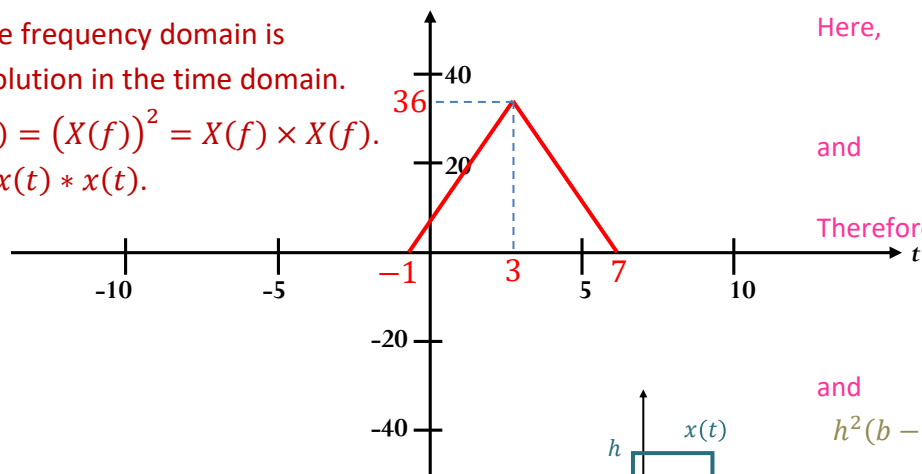2. Define another signal $y(t)$ whose Fourier transform is given by

$$Y(f) = (X(f))^2.$$

Plot $y(t)$.

Multiplication in the frequency domain is equivalent to convolution in the time domain.

Here, we have $Y(f) = (X(f))^2 = X(f) \times X(f)$.
Therefore, $y(t) = x(t) * x(t)$.

40
36
20
-10   -5 -1 3   5   7   10 t
-20
-40

In class, we have shown that when we have a rect. function,
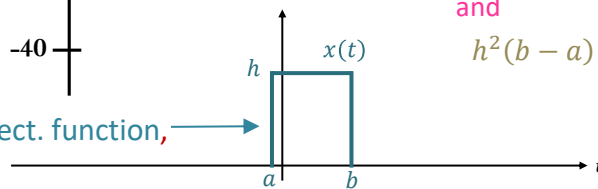
its self-convolution is a triangular func.

Here,
$a = -0.5$,
$b = 3.5$,
and
$h = 3$.
Therefore,
$2a = -1$,
$2b = 7$,
$a + b = 3$,
and
$h^2(b - a) = 3^2(3.5 - (-0.5))$
$= 9 \times 4 = 36$.

h $x(t)$

a b t

$h^2(b - a)$

$2a$ $2b$ t

$t = a + b$

$$x(t) * x(t) = \int_{-\infty}^{\infty} x(\tau)x(t - \tau)d\tau$$

1. Consider a modulator below.

$m(t)$

Message
(modulating signal)

$A_1 \cos(2\pi f_c t)$

Modulator

$x(t)$

Recall that, when $X(f) = m(t)A_1 \cos(2\pi f_c t)$, we know that

$$X(f) = \frac{A_1}{2}M(f - f_c) + \frac{A_1}{2}M(f + f_c).$$

Suppose $A_1 = 2$, f_c = 40 Hz, and the Fourier transform of the message is as plotted below.

Here, $A_1 = 2$ and $f_c = 40$. Therefore,

Plot $X(f)$ in the corresponding space below.

$$X(f) = M(f + 40) + M(f - 40).$$

$M(f)$

3

-20   10 f

$X(f)$

3

-100   -60   -30   20   50   100 f

2. Consider two signals $m(t)$ and $g(t)$.

   The magnitude plots of their Fourier transforms are shown below.

$|M(f)|$

2

8   f

$|G(f)|$

1

20   f
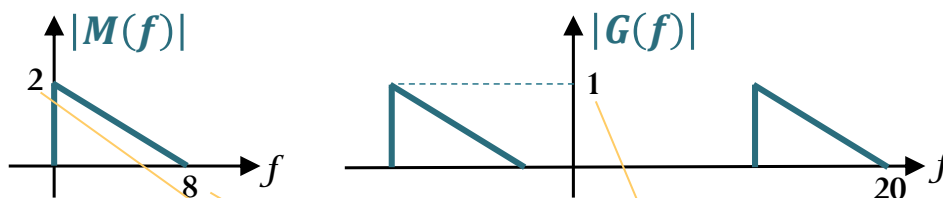
In the time domain, suppose $g(t) = c_1 m(t)\cos(c_2 t)$ for some positive constants c_1 and c_2.

Find the values of the constants c_1 and c_2:

$$c_1 = \underline{\quad 1 \quad}, c_2 = \underline{\quad 24\pi \quad}.$$

From $g(t) = c_1 m(t) \cos(c_2 t) = c_1 m(t) \cos\left(2\pi \frac{c_2}{2\pi} t\right)$, we know that $G(f) = \frac{c_1}{2}M\left(f - \frac{c_2}{2\pi}\right) + \frac{c_1}{2}M\left(f + \frac{c_2}{2\pi}\right)$.

$$\frac{c_1}{2} \times 2 = 1 \qquad 8 + \frac{c_2}{2\pi} = 20$$

$$c_1 = 1 \qquad c_2 = 24\pi$$

ECS 332: In-Class Exercise # 6 - Sol

Date: 1 1 / 0 9 / 2019
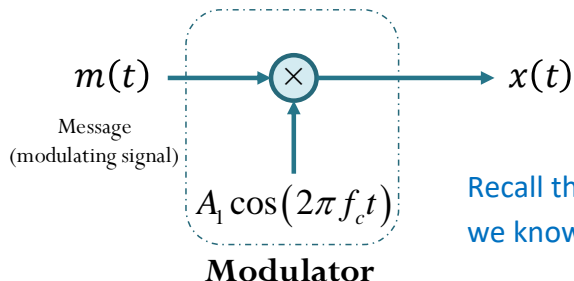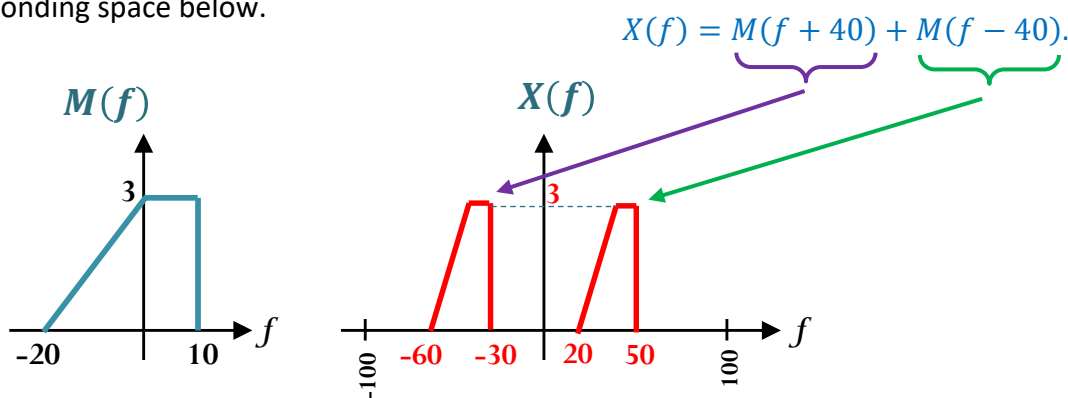
Name		ID (last 3 digits)	

Instructions

1. Separate into groups of no more than three students each.
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2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

In this problem, we have three "devices".

- $\boxed{(\,\cdot\,)^2}$ is a "square" device. As the name suggests, its output is created by squaring its input in the **time** domain.

- $\boxed{H_1(f)}$ is an LTI device whose **frequency response** is $H_1(f) = \begin{cases} 1, & |f| < 234, \\ 0, & \text{otherwise.} \end{cases}$

- $\boxed{H_2(f)}$ is an LTI device whose **frequency response** is $H_2(f) = \begin{cases} 1, & |f| > 234, \\ 0, & \text{otherwise.} \end{cases}$

★ Recall that

$$e^{j2\pi f_0 t} \to \boxed{H(f)} \to H(f_0)e^{j2\pi f_0 t}$$

Find the output $y(t)$ for each of the systems below.

$j2\pi f_0 t = 332\pi t \Rightarrow f_0 = 166$

(a) $x(t) = e^{332\pi t} \longrightarrow \boxed{H_1(f)} \longrightarrow y(t)$

$H_1(166) = 1$ because $|166| < 234$.

$y(t) = H_1(f_0)e^{j2\pi f_0 t} = H_1(166)e^{j2\pi(166)t} = 1e^{j332\pi t} = 1e^{j332\pi t}$

◇ Recall that

(b) $x(t) = \cos(332\pi t) \longrightarrow \boxed{H_1(f)} \longrightarrow y(t)$

$$\cos(2\pi f_0 t) \to \boxed{H(f)} \to \frac{1}{2}H(f_0)e^{j2\pi f_0 t} + \frac{1}{2}H(-f_0)e^{-j2\pi f_0 t}$$

$$= H(f_0)\cos(2\pi f_0 t)$$

$y(t) = H_1(f_0)\cos(2\pi f_0 t) = \overbrace{H_1(166)}^{1}\cos(2\pi(166)t) = \cos(332\pi t)$

when $H(f)$ is an even function which is the case here

(c) $x(t) = \cos(332\pi t) \longrightarrow \boxed{H_2(f)} \longrightarrow y(t)$

$y(t) = H_2(f_0)\cos(2\pi f_0 t) = \overbrace{H_2(166)}^{0}\cos(2\pi(166)t) = 0$

One can view the constant $\frac{1}{2}$ as a complex-expo. function $\frac{1}{2}e^{j2\pi(0)t}$ whose freq. is 0

(d) $x(t) = \cos(332\pi t) \longrightarrow \boxed{(\,\cdot\,)^2} \xrightarrow{x^2(t)} \boxed{H_1(f)} \longrightarrow y(t)$

$$x^2(t) = \cos^2(332\pi t) = \left(\frac{e^{j332\pi t} + e^{-j332\pi t}}{2}\right)^2 = \frac{1}{4}e^{j2\pi(332)t} + \frac{1}{2} + \frac{1}{4}e^{j2\pi(-332)t}$$

So, $x^2(t)$ is simply a linear combination of complex-exponential functions. Therefore, we can apply our ★ to each term.

$$y(t) = \frac{1}{4}\underbrace{H_1(332)}_{0}e^{j2\pi(332)t} + \frac{1}{2}\overbrace{H_1(0)}^{1} + \frac{1}{4}\underbrace{H_1(-332)}_{0}e^{j2\pi(-332)t} = \frac{1}{2}$$

(e) $x(t) = \cos(332\pi t) \longrightarrow \boxed{(\,\cdot\,)^2} \xrightarrow{x^2(t)} \boxed{H_2(f)} \longrightarrow y(t)$

Here, we can still use the expression of $x^2(t)$ derived in the previous part. However, we have to change the frequency response of the device from $H_1(f)$ to $H_2(f)$.

$$y(t) = \frac{1}{4}\underbrace{H_2(332)}_{1}e^{j2\pi(332)t} + \frac{1}{2}\underbrace{H_2(0)}_{0} + \frac{1}{4}\underbrace{H_2(-332)}_{1}e^{j2\pi(-332)t} = \frac{1}{4}e^{j2\pi(332)t} + \frac{1}{4}e^{j2\pi(-332)t} = \frac{1}{2}\cos(664\pi t)$$

ECS 332: In-Class Exercise # 7 - Sol

Instructions

1. Separate into groups of no more than three students each.
   The group cannot be the same as any of your former groups.
2. Explanation is not required for this exercise [ENRE]
3. **Do not panic.**

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1. Consider an LTI communication channel.
   Suppose when we put

   $$x(t) = 4\cos(\pi t) + 2\cos(2\pi t) + \cos(4\pi t) + 0.5\cos(6\pi t) + \cos(8\pi t) + 1$$

   into this channel, we get

   $$y(t) = 0.5\cos(\pi t) + e^{j2\pi t} + 2\sin(4\pi t) + 4\cos(6\pi t) + 8$$

   as its output.

   Let $H(f)$ be the frequency response of the channel that satisfies the above input-output relation.

   Find the value of $H(f)$ at each of the frequencies in the table below.

f	-3	-1	1	2	3
$H(f)$	8	0	1	$-2j$	8

In class, we have seen two properties of LTI system:

- Suppose we know that

  $$ae^{j2\pi f_0 t} \rightarrow \boxed{H(f)} \rightarrow be^{j2\pi f_0 t}.$$

  Then, we can infer that

  $$H(f_0) = \frac{b}{a}.$$

- Suppose we know that

  $$a\cos(2\pi f_0 t) \rightarrow \boxed{H(f)} \rightarrow b\cos(2\pi f_0 t + \theta).$$

  Then, we can infer that

  $$H(f_0) = \frac{b}{a}e^{j\theta} \quad \text{and} \quad H(-f_0) = \frac{b}{a}e^{-j\theta}.$$

Here, we are given f_0. Hence, the key is to find which parts of the input $x(t)$ and output $y(t)$ correspond to the given frequency.

- For $f_0 = \pm 3$, we are given that

  $$0.5\cos(6\pi t) \rightarrow H(f) \rightarrow 4\cos(6\pi t + 0).$$

  Therefore, $H(3) = \frac{4}{0.5}e^{j0} = 8$ and $H(-3) = \frac{4}{0.5}e^{-j0} = 8$.

- For $f_0 = \pm 1$, we are given that

  $$2\cos(2\pi t) \rightarrow H(f) \rightarrow e^{j2\pi t}.$$

  This does not directly fit the known forms above. However, once we apply the Euler's formula, we get

  $$2\cos(2\pi t) = e^{j2\pi t} + e^{-j2\pi t}.$$

  Therefore, we are given that

  $$e^{j2\pi t} + e^{-j2\pi t} \rightarrow H(f) \rightarrow e^{j2\pi t} + 0e^{-j2\pi t}.$$

  Therefore, $H(1) = \frac{1}{1} = 1$ and $H(-1) = \frac{0}{1} = 0$.

- For $f_0 = \pm 2$, we are given that

  $$\cos(4\pi t) \rightarrow H(f) \rightarrow 2\sin(4\pi t).$$

  Recall that $\sin(x) = \cos(x - 90°)$. Therefore, we know that

  $$\cos(4\pi t) \rightarrow H(f) \rightarrow 2\cos(4\pi t - 90°).$$

  Therefore, $H(3) = \frac{2}{1}e^{j(-90°)} = 2(-j) = -2j.$

ECS 332: In-Class Exercise # 8 - Sol

Instructions

1. Separate into groups of no more than three students each.
   The group cannot be the same as any of your former groups.
2. Explanation is not required for this exercise [ENRE]
3. **Do not panic.**

Name	ID (last 3 digits)		
Prapun	5	5	5

1. The impulse response of a multipath channel is of the form

$$h(t) = \sum_{k=1}^{v} \beta_k \delta(t - \tau_k).$$

   a. Suppose $v = 2$, $\beta_1 = \beta_2 = 0.5$, $\tau_1 = 1$, $\tau_2 = 3$.

   For each of the following channel input $x(t)$, find the corresponding channel output $y(t)$.

   Note that the output should be of the form $y(t) = A\cos(2\pi f_0 t + \theta_0)$ for some constants A, f_0, and θ_0.

Channel input	Channel output
$x(t) = \cos(\pi t)$	$\begin{aligned} y(t) &= 0.5x(t-1) + 0.5x(t-3) \\ &= 0.5\cos(\pi(t-1)) + 0.5\cos(\pi(t-3)) \\ &= 0.5\cos(\pi t - \pi) + 0.5\cos(\pi t - 3\pi) \\ &= -0.5\cos(\pi t) - 0.5\cos(\pi t) \\ &= -\cos(\pi t) \end{aligned}$
$x(t) = \cos\left(\dfrac{\pi}{2}t\right)$	$\begin{aligned} y(t) &= 0.5x(t-1) + 0.5x(t-3) \\ &= 0.5\cos\left(\frac{\pi}{2}(t-1)\right) + 0.5\cos\left(\frac{\pi}{2}(t-3)\right) \\ &= 0.5\cos\left(\frac{\pi}{2}t - \frac{\pi}{2}\right) + 0.5\cos\left(\frac{\pi}{2}t - \frac{3\pi}{2}\right) \end{aligned}$ Conversion to phasor form $\Longleftrightarrow 0.5 \angle -90° + 0.5 \angle -270° = 0$ Conversion back to time domain $\Longleftrightarrow 0\cos\left(\frac{\pi}{2}t + 0\right) \equiv 0$

   b. Suppose $v = 1$, $\beta_1 = 0.5$, $\tau_1 = 3$.

   Plot $|H(f)|$ from $f = -1$ to $f = 1$ Hz.
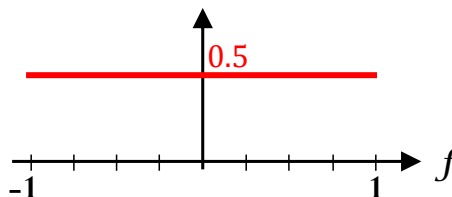
   When $v = 1$, we have $h(t) = \beta_1 \delta(t - \tau_1)$. With the provided values, we have

$$h(t) = 0.5\delta(t - 3).$$

   Therefore, $H(f) = 0.5e^{-j2\pi 3f}$ and $|H(f)| = 0.5|e^{-j6\pi f}| \equiv 0.5 \times 1 = 0.5$.

   Note that this is a distortionless channel. So, the magnitude spectrum should be flat.

ECS 332: In-Class Exercise # 9 - Sol

Instructions

1. Separate into groups of no more than three students each.
   The group cannot be the same as any of your former groups.
2. Explanation is not required for this exercise [ENRE]
3. **Do not panic.**

Date: <u>1 9</u>/ <u>0 9</u>/ 2019

Name	ID (last 3 digits)		
Prapun	5	5	5

1. For each of the following signal $g(t)$, find its (normalized) average power $P_g \equiv \langle |g(t)|^2 \rangle$.

   <u>Do not use any approximation.</u>

	$g(t)$	$P_g = \langle |g(t)|^2 \rangle$
Linear combination of complex exponential functions [4.23]	$\sum_k c_k e^{j2\pi f_k t}$ where the f_k are distinct	$\sum_k |c_k|^2$
Linear combination of sinusoids [4.28]	$\sum_k A_k \cos(2\pi f_k t + \phi_k)$ where the f_k are positive and distinct	$\frac{1}{2}\sum_k |A_k|^2$

$g(t)$	
$g(t) = 10e^{j20\pi t}$	$P_g = 10^2 = 100.$
$g(t) = 10e^{j20\pi t} + 5e^{j40\pi t}$	First, we check that the freq. of the two terms are different which is the case here. Therefore, $$P_g = 10^2 + 5^2 = 125.$$
$g(t) = \left(10e^{j20\pi t} + 5e^{j40\pi t}\right)^2$	$$g(t) = \left(10e^{j20\pi t}\right)^2 + 2\left(10e^{j20\pi t}\right)\left(5e^{j40\pi t}\right) + \left(5e^{j40\pi t}\right)^2$$ $$= 100e^{j40\pi t} + 100e^{j60\pi t} + 25e^{j80\pi t}.$$ These terms have different freq. Therefore, $$P_g = 100^2 + 100^2 + 25^2 = 20625.$$
$g(t) = 4\cos\left(4t + 4°\right)$	For sinusoidal signals, don't forget that we have an additional factor of $\frac{1}{2}$. $$P_g = \frac{1}{2} \times 4^2 = 8.$$
$g(t) = 5\cos\left(3t + 15°\right) + 12\cos\left(4t + 105°\right)$	First, we check that the freq. of the two terms are different and positive which is the case here. Therefore, $$P_g = \frac{1}{2} \times 5^2 + \frac{1}{2} \times 12^2 = 84.5.$$
$g(t) = 5\cos\left(3t + 15°\right) + 12\cos\left(3t + 105°\right)$	The freq. of the two terms are the same. Therefore, we must combine them first: $$g(t) \Leftrightarrow 5\angle 15° + 12\angle 105° = 13\angle 82.38°$$ $$\Leftrightarrow 13\cos(3t + 82.38°).$$ Therefore, $P_g = \frac{1}{2} \times 13^2 = 84.5.$

Note that we don't need this angle.
We only need the magnitude for our power calculation. Knowing that the angle difference between the two terms is 90°, we can use Pythagoras' theorem: $\sqrt{5^2 + 12^2} = 13$.

Instructions

Name		ID (last 3 digits)	
Prapun		5	5	5

1. Separate into groups of no more than three students each.
   The group cannot be the same as any of your former groups.
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
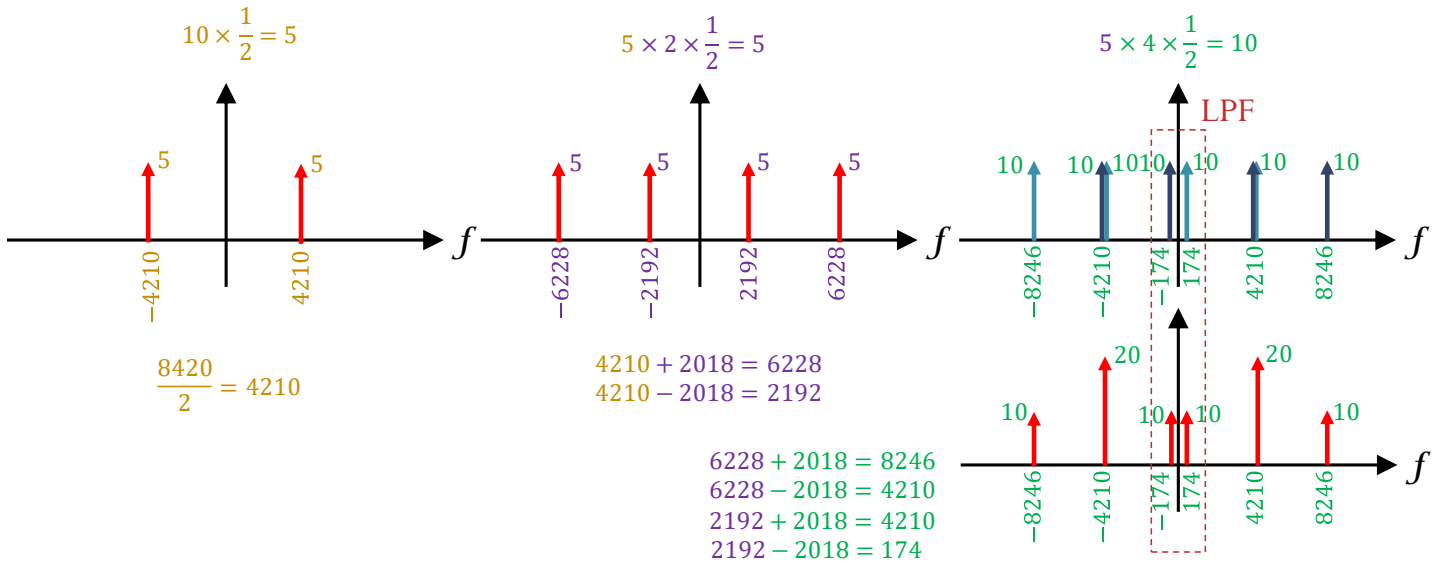3. **Do not panic.**

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1. Consider a DSB-SC modem with no channel impairment shown below.
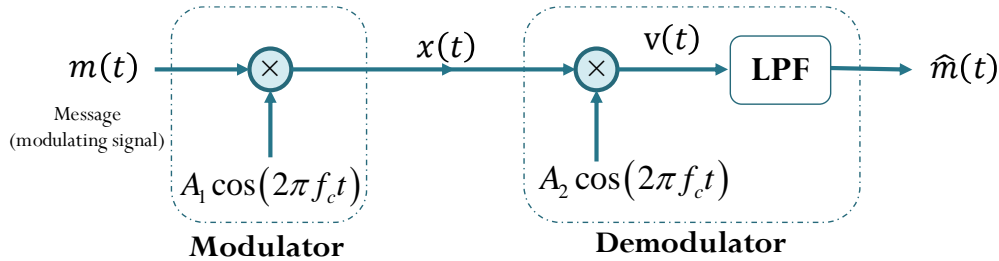
Let $A_1 = 2$, $A_2 = 4$, and $f_c = 2018$ Hz. Suppose LPF has $H_{LP}(f) = \begin{cases} 1, & |f| \leq 332, \\ 0, & \text{otherwise.} \end{cases}$

Suppose $m(t) = 10\cos(8420\pi t)$.

a) Plot $M(f), X(f),$ and $V(f)$.

$10 \times \dfrac{1}{2} = 5$

$\dfrac{8420}{2} = 4210$

$5 \times 2 \times \dfrac{1}{2} = 5$

$4210 + 2018 = 6228$
$4210 - 2018 = 2192$

$5 \times 4 \times \dfrac{1}{2} = 10$

$6228 + 2018 = 8246$
$6228 - 2018 = 4210$
$2192 + 2018 = 4210$
$2192 - 2018 = 174$

b) Find the corresponding $\hat{m}(t)$.

In the freq. domain, we have $\widehat{M}(f) = H_{LF}(f)V(f)$. Using $V(f)$ from part (a), eliminating the terms at $|f| > 332$, we get

$$\widehat{M}(f) = 10\delta(f - (-174)) + 10\delta(f - 174).$$

Therefore,

$$m(t) = 10e^{j2\pi(-174)t} + 10e^{j2\pi(174)t} = 20\cos(348\pi t).$$

Alternatively, we can work in the time domain directly.

We will need to use a trigonometric identity that we have proved in our HW:

$$\cos(A)\cos(b) = \frac{1}{2}\left(\cos(A+B) + \cos(A-B)\right).$$

Here, we have

$$v(t) = x(t) \times 4\cos(2\pi(2018)t) = \left(m(t) \times 2\cos(2\pi(2018)t)\right) \times 4\cos(2\pi(2018)t)$$

$$= m(t) \times 8\cos^2(2\pi(2018)t) = m(t) \times 4(1 + \cos(2\pi(2 \times 2018)t))$$

$$= 4m(t) + 4m(t)\cos(2\pi(2 \times 2018)t).$$

Substituting $m(t) = 10\cos(8420\pi t) = 10\cos(2\pi(4210)t)$, we get

$$v(t) = 40\cos(2\pi(4210)t) + 40\cos(2\pi(4210)t)\cos(2\pi(4036)t)$$

$$= 40\cos(2\pi(4210)t) + 20\cos(2\pi(174)t) + 20\cos(2\pi(8246)t).$$

After the LPF, only sinusoid with freq. not exceeding 332 passes through.

Therefore,

$$m(t) = 20\cos(348\pi t).$$