Name ________________________ ID3 _ _ _

ECS 332: Principles of Communications **2019/1**

HW 6 — Due: October 23, 4 PM

Lecturer: Prapun Suksompong, Ph.D.

Instructions

(a) This assignment has 2 pages.

(b) (1 pt) Hard-copies are distributed in class. Original pdf file can be downloaded from the course website. Work and write your answers **directly on the provided hardcopy/file** (not on other blank sheet(s) of paper).

(c) (1 pt) Write your first name and the last three digits of your student ID in the spaces provided on the upper-right corner of this page. Furthermore, for online submission, your file name should start with your 10-digit student ID, followed by a space, the course code, a space, and the assignment number: "5565242231 332 HW6.pdf"

(d) (8 pt) Try to solve all problems.

(e) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Problem 1. Consider the impulse train $G(f)$ shown on the right in Figure 6.1. Plot $g(t)$.

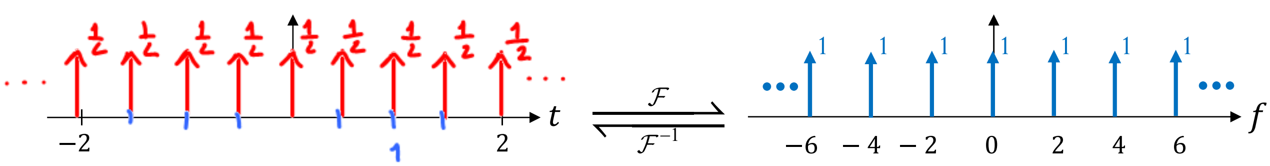


Figure 6.1: A train of impulses in the frequency domain

In class, we have seen that

$$\sum_{k=-\infty}^{\infty} \delta(t-kT_0) \xrightarrow{\mathcal{F}} \sum_{k=-\infty}^{\infty} f_0\,\delta(f-kf_0) \quad \text{where } f_0 = \frac{1}{T_0}.$$

If we use only this fact, then from the size of the δ-functions in $G(f)$, we have $f_0 = 1$.

However, from the spacing btw adjacent δ-func., we have $f_0 = 2$ ⇒ contradiction.

So, we know that the size of the δ-func. in the time domain is not "1".

By scaling the size of the δ-func. in the time domain by A, we have

$$(*) \qquad \sum_{k=-\infty}^{\infty} A\,\delta(t-kT_0) \xrightarrow{\mathcal{F}} \sum_{k=-\infty}^{\infty} Af_0\,\delta(f-kf_0).$$

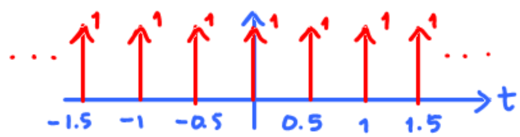
From $G(f)$, spacing gives $f_0 = 2$. $T_0 = \frac{1}{2}$

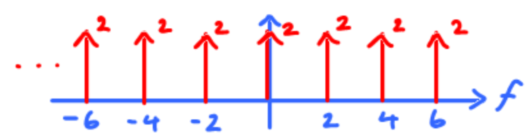
Size gives $Af_0 = 1 \Rightarrow A = \frac{1}{2}$

Problem 2. Consider a signal $r(t) = \sum_{k=-\infty}^{\infty} 2e^{j4\pi kt}$. Plot $r(t)$ and $R(f)$.
Hint: Don't try to actually plot each complex-expo. func. and add them. It is quite hopeless to determine their combination.

$r(t) = \sum_{k=-\infty}^{\infty} 2e^{j2\pi(k2)t} \xrightarrow{\mathcal{F}} R(f) = \sum_{k=-\infty}^{\infty} 2\delta(f - kf_0)$ where $f_0 = 2 \Rightarrow T_0 = \frac{1}{2}$

Using (*) from Problem 1, we have $Af_0 = 2 \Rightarrow A = 1$.





Problem 3. Consider a "square" wave (a train of rectangular pulses) shown in Figure 6.2. Its values periodically alternates between two values A and 0 with period T_0. At $t = 0$, its value is A.

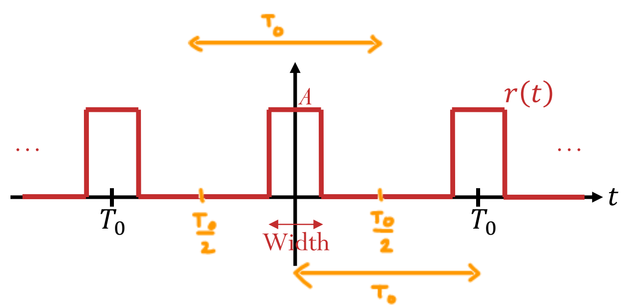


Figure 6.2: A train of rectangular pulses

First, recall that

duty cycle $= \frac{\text{width}}{T_0}$

Some values of its Fourier series coefficients are provided in the table below:

k	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7
c_k	$-\frac{\sqrt{2}}{7\pi}$	$-\frac{1}{3\pi}$	$-\frac{\sqrt{2}}{5\pi}$	0	$\frac{\sqrt{2}}{3\pi}$	$\frac{1}{\pi}$	$\frac{\sqrt{2}}{\pi}$	$\frac{1}{2}$	$\frac{\sqrt{2}}{\pi}$	$\frac{1}{\pi}$	$\frac{\sqrt{2}}{3\pi}$	0	$-\frac{\sqrt{2}}{5\pi}$	$-\frac{1}{3\pi}$	$-\frac{\sqrt{2}}{7\pi}$

(a) Find its duty cycle.

In class, we've seen that when the duty cycle is $\frac{1}{n}$, the n^{th} harmonic (along with its multiples) is suppressed.

Here, $c_4 = 0$. So, we conclude that the duty cycle is $\frac{1}{4} = 25\%$.

(b) Find the value of A. (Hint: Use c_0.)

Recall that $c_0 = \frac{1}{T_0}\int_{T_0} r(t)\,dt = \underbrace{\langle r(t)\rangle}_{\text{time average}}$.

From the picture, $\langle r(t)\rangle = \frac{\text{width} \times A}{T_0} = (\text{duty cycle}) \times A$. Therefore, $A = \frac{\langle r(t)\rangle}{\text{duty cycle}}$

We are given that $c_0 = \frac{1}{2}$ and we found, in part (a), that duty cycle $= \frac{1}{4}$.

Therefore, $A = \frac{1/2}{1/4} = 2$.