Name ________________________ ID3 _ _ _

ECS 332: Principles of Communications **2019/1**

HW 10 — Due: Not Due **Solution**

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Problem 1. State the Nyquist's (first) criterion for zero ISI

(a) In the time domain.

$$p(t) = \begin{cases} 1, & t = 0 \\ 0, & t = \pm T, \pm 2T, \pm 3T, \dots \end{cases}$$

symbol "duration" "interval"

(b) In the frequency domain.

$$\star \quad \sum_{k=-\infty}^{\infty} P\left(f - \frac{k}{T}\right) \equiv T$$

Reminder: A pulse $p(t)$ is called a Nyquist pulse iff it satisfies $\star$

Problem 2. In each part below, a pulse $P(f)$ is defined in the frequency domain from $f = 0$ to $f = 1$. Outside of $[0, 1]$, your task is to assign value(s) to $P(f)$ so that it becomes a Nyquist pulse. Of course, you will also need to specify the symbol interval T as well.

Hint: To avoid dealing with complex-valued $P(f)$, you may assume that $p(t)$ is real-valued and even; in which case $P(f)$ is also real-valued and even.

General strategy (recipe): Let the end of the given interval be $\frac{1}{2T}$.

These are chosen to ease the design. (will return to explain them later.)

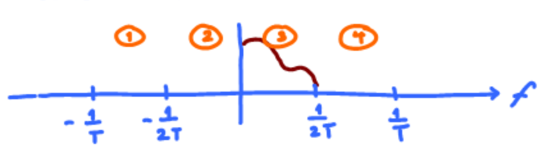
Then, set the symbol duration to be $T = \frac{1}{2f_0}$.

Our pulse will be band-limited to $\frac{1}{T}$.

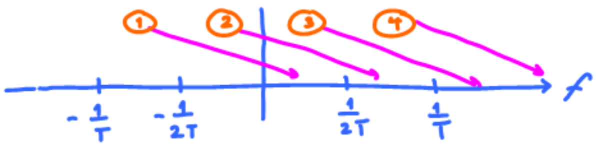
Recall that to check whether a pulse is a Nyquist pulse, we check whether $\sum_{k=-\infty}^{\infty} P\left(f - \frac{k}{T}\right) \equiv T$.

$P(f)$ is replicated every $\frac{1}{T}$.

Now consider 4 intervals:



Note that when $P(f)$ is copied to $\frac{1}{T}$, its content in region ① will show up in region ③



Therefore, when we add $P(f)$ in region ① and $P(f)$ in region ③, we must have T.

In other words, $P(f)$ in region ① can be found by $T - P(f)$ in region ③.

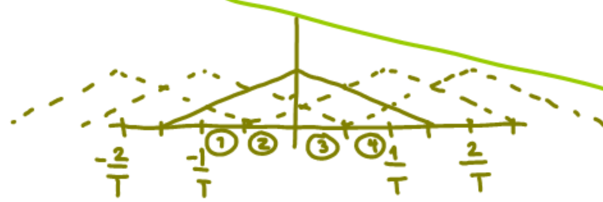
The suggested (hinted) symmetry in $P(f)$ allow us to find

$P(f)$ in region ② by flipping $P(f)$ in region ③ horizontally

and

$P(f)$ in region ④ by flipping $P(f)$ in region ① horizontally.

Remark: We don't want $P(f)$ to be non-zero outside region ①-④ because when we apply $\sum_{k=-\infty}^{\infty} P(f - \frac{k}{T})$, if $P(f)$ is too wide, we will have to deal with many overlapping replicas.



We don't want $P(f)$ to be zero inside region ④ (and hence ①) because we may need it to "cancel" the region ③ value to get the Σ to be T.

This is why we set the provided portion of $P(f)$ to be in region ③.

For this question, we have $f_0 = 1$. Therefore, we will choose

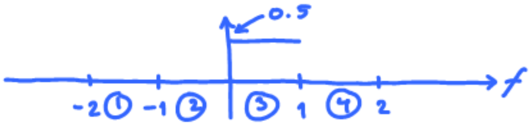
$T = \frac{1}{2f_0} = 0.5. \Rightarrow \frac{1}{T} = 2$ and $\frac{1}{2T} = 1.$

The four regions under consideration are
① $f \in [-2, -1)$
② $f \in [-1, 0)$
③ $f \in [0, 1)$
④ $f \in [1, 2]$

The question specifies $P(f)$ in region ③ as discussed above.

We will use the strategy described above to find $P(f)$ which is band-limited to $\frac{1}{T}$.

(a) Find a Nyquist pulse $P(f)$ whose $P(f) = 0.5$ on $[0, 1]$.



Note that in region 3, $P(f) \equiv T$ already.
Therefore, we need nothing from region 1;
in other words, $P(f) \equiv 0$ in region 1

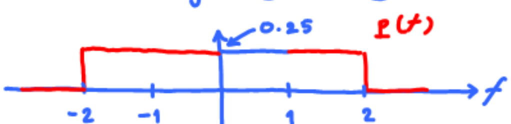
The values in region 2 and 4 are from the symmetry in $P(f)$.

(There is no need to check whether $P(f)$ satisfies the Nyquist criterion again because it is constructed by our general recipe)

(b) Find a Nyquist pulse $P(f)$ whose $\underbrace{P(f) = 0.25 \text{ on } [0, 1]}_{\text{region } 3}$.

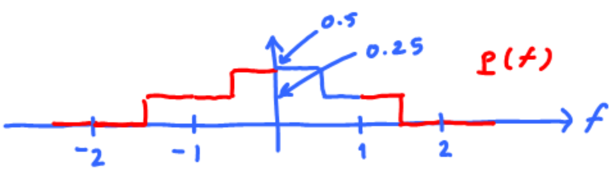
So, we will set the value in region 1 to be $T - 0.25 = 0.5 - 0.25 = 0.25$.

The values in region 2 and 4 are from the symmetry in $P(f)$.



(c) Find a Nyquist pulse $P(f)$ whose

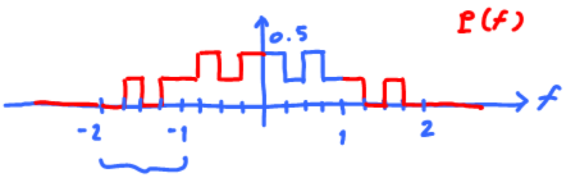
$$P(f) = \begin{cases} 0.5, & 0 \le f < 0.5 \\ 0.25, & 0.5 \le f \le 1 \end{cases}$$



As before, the value in region 1 is $T -$ region 3 $= 0.5 -$  $=$ .

(d) Find a Nyquist pulse $P(f)$ whose

$$P(f) = \begin{cases} 0.5, & f \in [0, 0.25) \cup [0.5, 0.75) \\ 0.25, & f \in [0.25, 0.5) \cup [0.75, 1] \end{cases}$$



$T -$ region 3 $= 0.5 -$  $=$ 

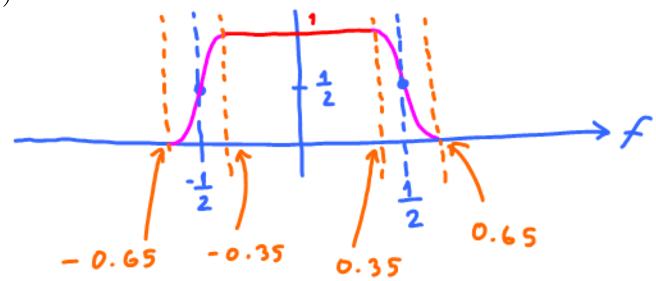
Problem 3. Consider a raised cosine pulse $p_{\mathrm{RC}}(t;\alpha)$ and its Fourier transform $P_{\mathrm{RC}}(f;\alpha)$. Assume the rolloff factor $\alpha = 0.3$ and the symbol "duration" $T = 1$.

(a) Carefully sketch $P_{\mathrm{RC}}(f;\alpha)$.

$\frac{1}{2T} = \frac{1}{2}$

$\frac{\alpha}{T} = \frac{0.3}{1} = 0.3$

$\frac{\alpha}{2T} = \frac{0.3}{2} = 0.15$



(b) Find $p_{\mathrm{RC}}(2;\alpha)$. The raised cosine pulse is a Nyquist pulse.

Therefore,
$$p(t) = \begin{cases} 1, & t = 0 \\ 0, & t = \pm T, \pm 2T, \pm 3T, \ldots \end{cases}$$

Here, $T = 1$.

Hence,
$$p(t) = \begin{cases} 1, & t = 0 \\ 0, & t = \pm 1, \pm 2, \pm 3 \ldots \end{cases}$$

In particular, $p(2) = 0$

(c) Find $P_{\mathrm{RC}}(0.5;\alpha)$.

Note that $P\left(\frac{1}{2T}\right) = \frac{T}{2}$. Here, $T = 1$. Hence, $P\left(\frac{1}{2}\right) = \frac{1}{2}$.

(d) Find $P_{\mathrm{RC}}(0.3;\alpha)$.

From part (a), we've seen that $P(f) = 1$ for $f \in [-0.35, 0.35]$. Therefore, $P(0.3) = 1$.

(e) *Find $P_{\mathrm{RC}}(0.4;\alpha)$.

We magnify the plot in part (a) for clarity

This is the "raised" part of RC.

$$P(0.4) = \frac{1}{2}\left(1 + \cos\left(\pi \times \frac{0.05}{0.3}\right)\right)$$

$$= \frac{1}{2}\left(1 + \cos\left(\pi/6\right)\right)$$

$$= \frac{2+\sqrt{3}}{4} \approx 0.933$$

Remark: You should be able to solve this problem without referring to the "ugly" expression (**??**) below.

f_s

Problem 4 (F2016,Q12). In a PCM system, an analog message is sampled at 5,000 [Sa/s] and then uniformly quantized into 128 different levels.
L

$\ell = 7$

(a) What is the (theoretical) maximum frequency of the analog message that can be used under such system without aliasing?

$\frac{5000}{2} = 2500 = 2.5$ [kHz]

(b) Calculate the bit rate of this system.

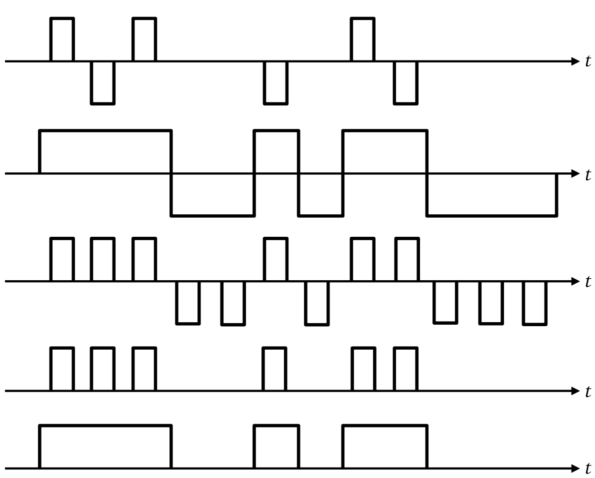
$R_b = f_s \times \ell = 5000 \times 7 = 35$ [kbps]

(c) Determine the signal-to-quantization-noise power ratio in dB when the message is a full-load sinusoidal modulating wave.

$1.76 + \underbrace{6\ell}_{42} = 43.76$ [dB]

Alternatively, $1.5L^2 = 24,576$
$= 43.9$ [dB]

Problem 5 (F2016, Q14). For each of the plots below, indicate which type of line code is being used. Put your answer in the provided box to the right of each plot. In all cases, the bit sequence used is 1 1 1 0 0 1 0 1 1 0 0 0.



Bipolar RZ

Polar NRZ

Polar RZ

Unipolar RZ

Unipolar NRZ

Possible answers are "Unipolar RZ", "Unipolar NRZ", "polar RZ", "polar NRZ", "bipolar RZ", "bipolar NRZ", "Manchester"

Extra Question

Here is an optional question for those who want more practice.

Problem 6. Consider a raised cosine pulse $p_{\mathrm{RC}}(t;\alpha)$ with rolloff factor α and symbol "duration" T. Its time domain expression is

$$p_{\mathrm{RC}}(t;\alpha) = \frac{\cos\frac{\alpha\pi t}{T}}{1-\frac{4\alpha^2t^2}{T^2}}\operatorname{sinc}\frac{\pi t}{T} = \frac{\cos\frac{\alpha\pi t}{T}}{1-\frac{4\alpha^2t^2}{T^2}}\frac{\sin\frac{\pi t}{T}}{\frac{\pi t}{T}}. \tag{10.1}$$

(a) Find $p(T/2)$ as a function of α.

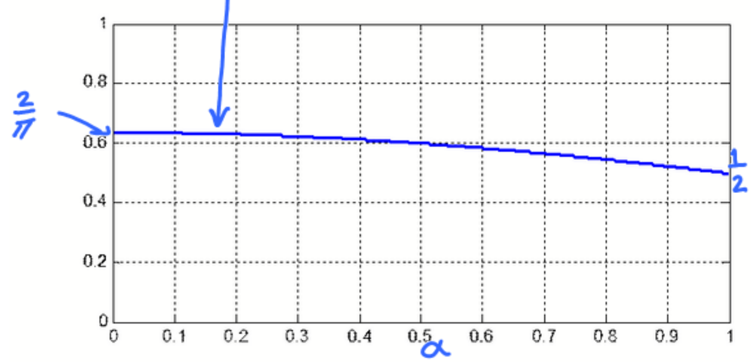
At $t = \frac{T}{2}$, we have $\frac{t}{T} = \frac{1}{2}$

$$p_{RC}\left(\tfrac{T}{2};\alpha\right) = \frac{\cos(\alpha\frac{\pi}{2})}{1-\alpha^2}\,\frac{\underbrace{\sin\frac{\pi}{2}}_{\to 1}}{\frac{\pi}{2}}$$

$$= \frac{2}{\pi}\times\frac{\cos(\alpha\frac{\pi}{2})}{1-\alpha^2}$$

(b) Use `MATLAB` to plot $p(T/2)$ as a function of α.

(c) Find $\lim_{\alpha\to 1} p_{\mathrm{RC}}\left(\frac{T}{2};\alpha\right)$.



Note that when $\alpha = 0$, $p_{RC}\left(\frac{T}{2};\alpha\right) = \frac{2}{\pi} \approx 0.6366$

As $\alpha \to 1$, $p_{RC}\left(\frac{T}{2};\alpha\right) \to \cancel{\frac{2}{\pi}} \times \left.\frac{\cancel{\frac{\pi}{2}}\left(-\sin(\alpha\frac{\pi}{2})\right)}{-2\alpha}\right|_{\alpha=1} = \frac{1}{2}.$